Representation Theory of Groups, Lie Algebras, and Hopf Algebras
Théorie de représentation des groupes, des algèbres de Lie et de Hopf
(Org: **Allen Herman** and/et **Fernando Szechtman** (Regina))

MURRAY BREMNER, University of Saskatchewan
The fundamental invariants of 3 x 3 x 3 arrays

We use computer algebra to determine explicitly the three fundamental invariants of a $3 \times 3 \times 3$ array over $\mathbb{C}$ as polynomials in the 27 variables x_{ijk} for $1 \leq i, j, k \leq 3$. By the work of Vinberg on θ-groups, it is known that these invariants are homogeneous polynomials of degrees 6, 9 and 12 with respectively 1152, 9216 and 209061 terms. These three polynomials freely generate the algebra of invariants for $\mathfrak{sl}(3, \mathbb{C})^3$ acting irreducibly on its natural representation $(\mathbb{C}^3)^{\otimes 3}$. We find compact expressions for these invariants in terms of the orbits of the finite group $(S_3 \times S_3 \times S_3) \rtimes S_3$ acting on monomials of weight zero. It remains an open problem to express the hyperdeterminant of degree 36 (in the sense of Gelfand et al.) in terms of these fundamental invariants. (This is joint work with Jiaxiong Hu.)

LEANDRO CAGLIERO, FAMAF - CONICET
The classification of uniserial $\mathfrak{sl}(2) \ltimes V(m)$-modules and a new interpretation of the Racah-Wigner $6j$-symbol

All Lie algebras and representations are assumed to be finite dimensional over $\mathbb{C}$. Let $V(m)$ be the irreducible $\mathfrak{sl}(2)$-module with highest weight $m \geq 1$ and let $\mathfrak{g}_m = \mathfrak{sl}(2) \ltimes V(m)$. In this talk we present a joint work with F. Szechtman in which we classify of all uniserial $\mathfrak{g}_m$-modules. Recall that a $\mathfrak{g}$-module is uniserial when its submodules form a chain. Uniserial modules are usually viewed as building blocks to understand more general classes of indecomposable representations. A classification of the indecomposable $\mathfrak{g}_m$-modules is far from being achieved even for $m = 1$, see [DR],[Pi].

In our classification, the main family of uniserial $\mathfrak{g}_m$-modules is actually constructed for any $\mathfrak{g} = \mathfrak{s} \ltimes V(\mu)$, where $\mathfrak{s}$ is a semisimple Lie algebra and $V(\mu)$ is the irreducible $\mathfrak{s}$-module with highest weight $\mu \neq 0$. It turns out that the members of this family are, but for a few exceptions of lengths 2, 3 and 4, the only uniserial $\mathfrak{g}_m$-modules.

One major step towards this classification is the determination of all admissible sequences of length 3, these are sequences $V(a), V(b), V(c)$ for which there is a uniserial $\mathfrak{g}_m$-module with these composition factors. This step depends in an essential manner on the determination of certain non-trivial zeros of Racah-Wigner $6j$-symbol.

References

[DR] A. Douglas, J. Repka, *Embedding of the Euclidean algebra e(3) into sl(4,C) and restriction of irreducible representations of sl(4,C)*, Journal of Mathematical Physics **52** 013504 (2011).

[Pi] A. Piard, *Sur des représentations indécomposables de dimension finie de $SL(2).R^2$*, Journal of Geometry and Physics, Volume **3**, Issue 1, 1986, 1–53.

PAOLO CASATI, Department of Mathematics University of Milano Bicocca
Simple representations of simple Lie algebras remain Indecomposable restricted to some Abelian Subalgebras

In this talk we show that any finite dimensional irreducible representation of a complex simple Lie algebra of rank n remains indecomposable if restricted to some abelian subalgebras of the (minimal as it will be explained in the talk) dimension n, extending the corresponding result obtained in [1] (Theorem 3.9) for the simple Lie algebra of type A_n. Such abelian subalgebra $\mathfrak{a}$ can be constructed as follows.

Let $\mathfrak{g}$ be the complex simple Lie algebra, $\mathfrak{h} \subset \mathfrak{g}$ its Cartan subalgebra and $\Delta = \Delta(\mathfrak{g}, \mathfrak{h})$ the corresponding set of roots. Further for any $\alpha \in \Delta$ let X_α be a basis of root space $\mathfrak{g}_\alpha = \{X \in \mathfrak{g} | \ [H, X] = \alpha(H)X \ \forall H \in \mathfrak{h}\}$, $\Pi = \{\alpha_1, \ldots, \alpha_n\}$ a set of simple roots in Δ and set $Y_{\alpha_i} = X_{-\alpha_i}$, then $\mathfrak{a}$ is the abelian subalgebra of $\mathfrak{g}$ spanned by the vectors $\{Y_{\alpha_{2i+1}}\}$ $(i = 0, \ldots, \left[\frac{n}{2}\right])$ and $\{X_{\alpha_{2j}}\}$ $(j = 1, \ldots, \left[\frac{n}{2}\right])$, where $[x]$ denotes the integer part of x.

[1] P. Casati *Irreducible SL_{n+1}–Representations remain Indecomposable restricted to some Abelian Subalgebras* Journal of Lie Theory Volume **20** (2010) 393–407

GERALD CLIFF, University of Alberta
Representations of GL(n,R) and Sp(2n,R) where R is a p-adic field or a finite local ring

Suppose that G is a reductive group defined over $\mathbb{Z}$, such as the general linear group GL_n or the symplectic group Sp_{2n}. Consider the representation space V of a smooth irreducible complex representation of $G(F)$ where F is a p-adic field. Restrict to $G(R)$ where R is the ring of integers of F. This gives rise to a series for V whose factors are sums of irreducible representations V_i of the finite groups $G(S_i)$ where S_i is R mod the i-th power of the maximal ideal of R. We discuss the interconnections between these representations V_i and the original representation V.

HUBERT DE GUISE, Lakehead University
Some indecomposable representations in physics

I will describe the construction and use of some indecomposable representations in physics. The Euclidean algebra $e(2)$ - one generator of rotation in the 2D plane plus two generators of translations in this plane - will be used as an example of some concepts. How this kind of construction can be generalized to the more physically important case of the Poincare algebra - 6 generators of spacetime rotations plus 4 generators of spacetime translations - remains in part an open problem.

HADER ELGENDY, University of Saskatchewan
The universal Associative envelope of the anti-Jordan triple system of $n \times n$ matrices

We show that the universal associative enveloping algebra of the simple anti-Jordan triple system of all $n \times n$ matrices $(n \geq 2)$ over an algebraically closed field of characteristic 0 is finite dimensional. We investigate the structure of the universal envelope and focus on the monomial basis, the structure constants, and the center. We explicitly determine the decomposition of the universal envelope into matrix algebras. We classify all finite dimensional irreducible representations of the simple anti-Jordan triple system, and show that the universal envelope is semisimple. We also provide an example to show that the universal enveloping algebras of anti-Jordan triple systems are not necessary to be finite-dimensional.

MICHAEL GELINE, Northern Illinois University
Knörr lattices for elementary abelian p-groups

An approach to Brauer's height zero conjecture due to Knörr leads to the study of a very rich family of p-adic representations of p-groups. These representations are called Knörr lattices. After outlining the approach to Brauer's conjecture, I will give a construction of positive height Knörr lattices for elementary abelian p-groups, and discuss bounds for the height in terms of the rank of the group. This is joint work with G. Robinson. Several open questions will be advertised.

NICOLAS GUAY, University of Alberta
Coideal subalgebras of quantized enveloping algebras.

Quantum groups, and in particular quantized enveloping algebras of semisimple Lie algebras, are Hopf algebras which have played a very important role in representation theory and mathematical physics in the past three decades. Certain twisted

Yangians and twisted quantum loop algebras can be realized as subalgebras of quantized enveloping algebras but are not themselves Hopf subalgebras: they are instead coideal subalgebras. A brief overview of some interesting coideal subalgebras of quantized enveloping algebras will be presented along with a few recent results for the twisted case of type AIII. (This talk will be partly based on joint work with Xiaoguang Ma.)

XIANGQIAN GUO, Zhengzhou University
New irreducible modules for Virasoro algebra

We construct two classes of irreducible Virasoro modules by twisting Harish-Chandra modules over the Heisenberg-Virasoro algebra using automorphisms of the Heisenberg-Virasoro algebra. Weight modules in the first class are some irreducible highest weight modules and non-weight modules in the first class are irreducible Whittaker modules. We also obtain concrete bases for these modules. This generalizes known results on Whittaker modules. The second class of modules are non-weight modules which are not Whittaker modules. We determine the irreducibility and isomorphism classes of these modules.

GERHARD HISS, RWTH Aachen University
On the Chevalley property of a finite-dimensional group algebra and its Drinfeld double

This is a report on joint work with Hui-Xiang Chen. A Hopf algebra has the Chevalley property, if the tensor product of any two simple modules is semisimple. Let G be a finite group and H the group algebra of G over some algebraically closed field. We give necessary and sufficient conditions on G guaranteeing that the Drinfeld double $D(H)$ has the Chevalley property. We also show that H and $D(H)$ have the Chevalley property if and only if the tensor product $V \otimes V^*$ is semisimple for every simple H-, respectively $D(H)$-module.

MICHAEL LAU, Université Laval
Representations of Twisted Forms

Various interesting rings can be viewed geometrically as twisted forms of current algebras. We will explain how to use this point of view to describe maximal ideals and classify simple modules of Azumaya algebras, affine Lie algebras and some of their higher-dimensional analogues. This talk is based on joint work with Arturo Pianzola.

MARK LEWIS, Kent State University
Counting characters in blocks of solvable groups with abelian defect group

If G is a solvable group and p is a prime, then the Fong-Swan theorem shows that given any irreducible Brauer character φ of G, there exists a character $\chi \in \mathrm{Irr}(G)$ such that $\chi^o = \varphi$, where ^{o} denotes the restriction of χ to the p-regular elements of G. We say that χ is a *lift* of φ in this case. It is known that if φ is in a block with abelian defect group D, then the number of lifts of φ is bounded above by $|D|$. In this paper we give a necessary and sufficient condition for this bound to be achieved, in terms of local information in a subgroup V determined by the block B. We also apply these methods to examine the situation when equality occurs in the $k(B)$ conjecture for blocks of solvable groups with abelian defect group.

This is joint work with J. P. Cossey.

SONIA NATALE, Universidad Nacional de Córdoba. CIEM-CONICET.
Extensions of tensor categories.

The talk will be based on joint work with A. Bruguières. We shall discuss normal tensor functors and exact sequences of tensor categories, which generalize Hopf algebra exact sequences and equivariantization under a finite group action on a tensor category. We shall present some applications of these notions to classification problems.

JOSE PANTOJA, Catholic University of Valparaiso
Weil Representations of the groups $GL^*_\epsilon(2,A)$ and $SL^*_\epsilon(2,A)$

Let A be a unitary ring with an involution $*$. Then the groups $GL^*_\epsilon(2,A)$ and $SL^*_\epsilon(2,A)$ are a (tamely) non-commutative version of the general linear and special linear groups over a field, consisting of 2×2 matrices with coefficients in A, that satisfy certain commuting relations which involve $*$. Symplectic groups, orthogonal groups and also non-classical groups are examples of the groups under consideration for different choices of the involutive ring.

Several times these groups afford Bruhat-like presentations. This is the case when A is an artinian simple involutive ring, and when A admits a weak $*$-analogue of the euclidean algorithm for coprime elements. A very general Weil representation can be constructed in this case, from abstract core data, (recovering as particular cases, the Weil representations of the symplectic groups $Sp(2n,k)$ for k a finite field and the generalized Weil representation of a non-classical case of an involutive base ring having a nilpotent radical).

When a presentation is not at hand a different but also elementary approach to the construction of Weil representations, which is more geometric in nature, can be applied.

RACHAEL QUINLAN, National University of Ireland Galway
Characters of covering groups of elementary abelian 2-groups

A covering group of the elementary abelian 2-group Q of rank n is a group G for which $G/G'\cong Q$ and $G'=Z(G)$ is elementary abelian of order $2^{\binom{n}{2}}$. Designating a particular covering group amounts G to writing the square of each of the n elements of a minimal generating set for G as a product of the $\binom{n}{2}$ simple commutators in the generators. One may investigate whether and when different such designations yield non-isomorphic covering groups. In this talk we discuss the question of how many characters of a covering group of Q may be real-valued, and describe up to isomorphism those groups in which the maximum possible number of real-valued characters is attained.

ANNA STOKKE, University of Winnipeg
Global crystal bases and q-Schur algebras

One approach to studying irreducible $U_q(sl_n)$-modules is through quantized Schur algebras, or q-Schur algebras. In this talk, I will give bases for irreducible $U_q(sl_n)$-modules in terms of q-Schur algebras which turn out to be quantum versions of the Carter-Lusztig standard bases. I will also establish the connection between these bases and Kashiwara's global crystal bases.

RYAN VINROOT, The College of William and Mary
Semisimple symplectic characters of finite unitary groups

Let $G=\mathrm{U}(2m,\mathbb{F}_q)$ be the finite unitary group defined over a finite field of order q, where q is the order of an odd prime p. We prove that the number of irreducible complex characters of G with degree coprime to p, and with Frobenius-Schur indicator -1, is equal to q^{m-1}. In particular, we find a (non-canonical) bijection between these irreducible characters and the set of self-dual polynomials of degree $2m$ over $\mathbb{F}_q$ with constant term -1. These results are joint work with Bhama Srinivasan, University of Illinois at Chicago.