Free Probability Theory: New Developments and Applications
Théorie des probabilités libres: applications et développements récents
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JAMES MINGO, Queen's University
Fluctuations of Free Wigner Matrices

A Wigner matrix is a self-adjoint (or symmetric) random matrix with i.i.d. entries above the diagonal. Their fluctuations were analyzed by Khorunzhy, Khoruzhenko, and Pastur. We consider the free analogue, i.e. freely independent non-commuting entries and analyze their fluctuations. Instead of non-crossing annular diagrams we get non-crossing linear diagrams. This is joint work with Roland Speicher.

RAJ RAO NADAKUDITI, University of Michigan
Numerical computation of convolutions in free probability

We develop a numerical approach for computing the additive, multiplicative and compressive convolution operations from free probability theory. We utilize the regularity properties of free convolution to identify (pairs of) 'admissible' measures whose convolution results in a so-called 'invertible measure' which is either a smoothly-decaying measure supported on the entire real line (such as the Gaussian) or square-root decaying measure supported on a compact interval (such as the semi-circle). This class of measures is important because these measures along with their Cauchy transforms can be accurately represented via a Fourier or Chebyshev series expansion, respectively. Thus knowledge of the functional inverse of their Cauchy transform suffices for numerically recovering the invertible measure via a non-standard yet well-behaved Vandermonde system of equations. We describe explicit algorithms for computing the inverse Cauchy transform alluded to and recovering the associated measure with spectral accuracy.

MIHAI V. POPA, Queen's University
Some Applications of Non-Commutative Functions in Free Analysis

Given two complex vector spaces, V and W, a non-commutative function is, briefly, a mapping from a certain class of subsets of that matrix space over V to the matrix space over W, satisfying some compatibility conditions: it has to respect direct sums and simultaneous similarities. Non-commutative functions have very strong regularity properties and they admit a very nice differential calculus, closely related to some QD-bialgebras arising in free probabilities. Such objects were considered before by J. L. Taylor in his groundbreaking work in on the noncommutative spectral theory, and more recently independently by D.-V. Voiculescu in free probability. Besides a brief introduction in the theory of non-commutative functions, the lecture will survey some applications of this theory in operator-valued non-commutative probability, such as non-commmutative Levy-Hincine formulas, Bercovici-Pata bijection, operator-valued Cauchy and R-transforms, operator-valued semicircle, arcsine and Bernoulli laws. Most of results presented are joint work with V. Vinnikov and Serban Belinschi.

EMILY REDELMEIER, Université Paris-Sud XI
Real Second-Order Freeness

The second-order statistics of large random matrices may be studied in a noncommutative probability space equipped with a bilinear function modelling the covariance of traces. As in the first-order case, second-order freeness may be defined so that fairly general classes of random matrices are asymptotically second-order free. However, the definition satisfied by real random matrices is different from that satisfied by their complex analogues. We present a topological approach to the matrix calculations, in which the real matrices are distinguished from their complex analogues by the appearance of twisted gluings and the resulting nonorientable surfaces. This motivates a different definition of second-order freeness in the real case, which is satisfied by a number of important matrix models, and in fact by any independent matrices which are orthogonally in general position.

NADIA SAAD, University of Ottawa
Integration of Invariant Matrices and Applications to Statistics

Expressing the local and global moments of random matrices is a common problem in different fields such as Random Matrix Theory, Statistics and Finance. Due to the important role played by moments of random matrices, we obtain an expression for the local moments of a number of complex and real matrices. A formula for the moments of left-right orthogonal (unitary) random matrices is our main result and it enables us to obtain the moments of the inverted Compound Wishart matrices. Since Covariance matrices with correlated samplings are Compound Wishart matrices, this explains the importance of the Compound Wishart matrices essentially in Statistics. This is joint work with Benoit Collins and Sho Matsumoto.

VICTOR VINNIKOV, Ben Gurion University of the Negev
Noncommutative Functions and Their Regularity Properties

Over the course of the last two decades, there emerged a general paradigm for passing from the commutative situation to the free noncommutative situation: we replace a vector space by the disjoint union of spaces of square matrices of all sizes over it. When applied to function theory on the vector space in question, this leads to noncommutative or fully matricial functions as studied by the speaker and D. S. Kaliuzhnyi-Verbovetskyi and by D.-V. Voiculescu; the origins of this theory actually go back to the pioneering work of J. L. Taylor on noncommutative functional calculi. In this talk, I will review some of the salient features of the theory of noncommutative functions with a special emphasis on their amazing regularity properties: over a finite-dimensional vector space, a noncommutative function that is locally bounded on slices, separately in every matrix dimension, is actually entrywise analytic in every matrix dimension, and admits a noncommutative power series expansion that converges locally uniformly.

JIUN-CHAU WANG, University of Saskatchewan
Questions around free Levy processes of the second kind

In a 1998 paper Biane started the investigation on the noncommutative stochastic processes with free increments, and he showed that there are two kinds of free Levy processes: the ones with stationary increment distributions (the first kind) and the ones with stationary transition probabilities (the second kind). In the literature, the second kind processes are less studied than are the first kind. In this talk we will explain briefly what is a free Levy process and then report some new results on the asymptotic distributional behavior for the marginal laws of a second kind free Levy process.