Let $G$ be a connected complex Lie group and $H$ a closed complex subgroup. There is a Lie theoretic fibration $\pi : G/H \to G/J$ with $G/J$ holomorphically separable and $\mathcal{O}(G/H) \simeq \pi^* \mathcal{O}(G/J)$ called the holomorphic reduction of the complex homogeneous manifold $G/H$. In general, $G/J$ is not Stein, e.g., $\mathbb{C}^n \setminus \{0\}$ for $n > 1$, and examples show that one need not have $\mathcal{O}(J/H) \simeq \mathbb{C}$.

We will prove that if $G/H$ is pseudoconvex and $G$ is reductive, then

1.) the base $G/J$ of its holomorphic reduction is Stein and $\mathcal{O}(J/H) \simeq \mathbb{C}$, and

2.) if additionally, $G/H$ is Kähler with $\mathcal{O}(G/H) \simeq \mathbb{C}$, then $G/H$ is a flag manifold, $\overline{H}/H$ is a Cousin group and $G/H = G/\overline{H} \times \overline{H}/H$ is a product, where $\overline{H}$ denotes the Zariski closure of $H$ in $G$.

The proof employs ideas of Hirschowitz (1975) in order to show the existence of a certain foliation of non-Stein pseudoconvex domains spread over complex homogeneous manifolds. This generalizes results of Kim-Levenberg-Yamaguchi (2011).

(Based on joint work with Christian Miebach and Karl Oeljeklaus.)