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Holomorphic Reductions of Pseudoconvex Homogeneous Manifolds

Let G be a connected complex Lie group and H a closed complex subgroup. There is a Lie theoretic fibration $\pi : G/H \to G/J$ with G/J holomorphically separable and $\mathcal{O}(G/H) \simeq \pi^*\mathcal{O}(G/J)$ called the holomorphic reduction of the complex homogeneous manifold G/H. In general, G/J is not Stein, e.g., $\mathbb{C}^n \backslash \{0\}$ for $n > 1$, and examples show that one need not have $\mathcal{O}(J/H) \simeq \mathbb{C}$.

We will prove that if G/H is pseudoconvex and G is reductive, then
1.) the base G/J of its holomorphic reduction is Stein and $\mathcal{O}(J/H) \simeq \mathbb{C}$, and
2.) if additionally, G/H is Kähler with $\mathcal{O}(G/H) \simeq \mathbb{C}$, then $G/\overline{H}$ is a flag manifold, $\overline{H}/H$ is a Cousin group and $G/H = G/\overline{H} \times \overline{H}/H$ is a product, where $\overline{H}$ denotes the Zariski closure of H in G.
The proof employs ideas of Hirschowitz (1975) in order to show the existence of a certain foliation of non-Stein pseudoconvex domains spread over complex homogeneous manifolds. This generalizes results of Kim-Levenberg-Yamaguchi (2011).
(Based on joint work with Christian Miebach and Karl Oeljeklaus.)