JUAN CARLOS BUSTAMANTE, Université de Sherbrooke

Hochschild cohomology and the derived class of $m$-cluster tilted algebras of type $\mathbb{A}$

Joint work with V. Gubitosi, from Sherbrooke, see http://arxiv.org/abs/1201.4182

For a given integer $m$ and a hereditary algebra $H$, the $m$-cluster category of $H$, $C_m(H)$ is obtained from the derived category $D(H)$ by identifying the Auslander-Reiten translation with the $m^{\text{th}}$ power of the shift $[1]^m := [m]$. In $C_m(H)$ there are cluster tilting objects, whose endomorphisms algebras are called $m$-cluster tilted algebras.

The aim of this work is to classify the algebras that are derived equivalent to $m$-cluster tilted algebras of type $\mathbb{A}$. The first result states that a connected algebra $A = kQ/I$ is derived equivalent to an $m$-cluster tilted algebra of type $\mathbb{A}$ if and only if it is gentle, having exactly $|Q_1| - |Q_0| + 1$ oriented cycles of length $m + 2$ each of which has full relations. We then prove:

Theorem: Let $A = kQ/I$ and $A' = k'Q'/I'$ be connected algebras derived equivalent to $m$-cluster tilted algebras of type $\mathbb{A}$. Then (among others) the following conditions are equivalent.

1. $A$ and $A'$ are derived equivalent,
2. $A$ and $A'$ are tilting-cotilting equivalent,
3. $\text{HH}^*(A) \simeq \text{HH}^*(A)$ and $K_0(A) \simeq K_0(A')$
4. $\pi_1(Q, I) \simeq \pi_1(Q', I')$ and $|Q_0| = |Q'_0|$. 

Our approach differs from previous works on related topics in the fact that we use the Hochschild cohomology ring as a derived invariant. We shall discuss about the proof and derive some consequences, among which we recover previous known results.