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On solution-free sets via local uniformity and energy incrementing

We consider a system of k diagonal polynomials of degrees $1, 2, \dots, k$. Using methods developed by W.T. Gowers and refined by Green and Tao to obtain bounds in the 4-term case of Szemeredi's Theorem on long arithmetic progressions, we show that if a subset $\mathcal{A}_N$ of the natural numbers up to N of size $\delta_N N$ exhibits sufficiently small local polynomial bias, then it furnishes roughly the expected number of solutions to the given system. If $\mathcal{A}_N$ furnishes no non-trivial solutions to the system, then we show via an energy incrementing argument that there is a concentration in a Bohr set of pure degree k, and consequently in a long arithmetic progression. We show that this leads to a bound on the density δ_N of the set $\mathcal{A}_N$ of the form $\delta_N \ll \exp(-c\sqrt{\log\log N})$, where $c > 0$ is a constant dependent at most on k.