Homotopy and Categories
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KRISTINE BAUER, University of Calgary
Cotriples, stabilization and Andre-Quillen homology

André-Quillen homology for commutative A-algebras over a fixed ring B has been well studied since its independent formulation by M. André and D. Quillen in the 1960's. On the one hand, André-Quillen homology is the left derived functor of the abelianization functor. On the other hand, André-Quillen homology is a special case of a cotriple homology theory (an observation of Barr). Topological André-Quillen homology (TAQ) was defined for E_∞ spectra by M. Basterra, and together with M. Mandell she showed that all homology theories of E_∞ A-algebras over a fixed spectrum B are André-Quillen homology (with coefficients). A consequence of this is that the stabilization of the forgetful functor U from E_∞ A-algebras over B to A-modules must be TAQ. On the other hand, in the special case $A = B$, M. Kantorovicz and R. McCarthy showed that the linear approximation of the forgetful functor from E_∞ A-algebras over A to A-modules is TAQ. Since the linearlization used by Kantorivicz-McCarthy comes from a cotriple construction, this result is akin to Barr's observation that the André-Quillen homology of a ring is a cotriple homology. At the time that Kantorovicz-McCarthy studied TAQ, the cotriple linearization was only available for functors from basepointed categories. In joint work with B. Johnson and R. McCarthy, we have extended the cotriple model of linearization to functors from categories which need not be basepointed. In this talk, I will explain the extended cotriple model of linearization and examine TAQ as a cotriple homology in the general setting.

RYAN BUDNEY, University of Victoria
What operads have to say about knots

This talk will be an overview of how certain operads relate to classical knot theory in the sense of the study of smooth embeddings of one sphere in another. There is an operad called "the splicing operad" which in dimension 3 gives an essentially complete description of the homotopy-type of the space of knots up to the solution of some rather subtle problems in 3-dimensional hyperbolic geometry. For embeddings of the circle in high-dimensional spheres, the splicing operad generates "cabling" operations which generate Turchin's "Hodge decomposition" for the Vassiliev spectral sequence for spaces of knots. It also gives rise to a rather unexpected connection between automorphisms of free groups and spaces of knots. In high dimensions there are also connections to algebraic k-theory via the work of Igusa, Hatcher, Farrell and Quillen.

ROBIN COCKETT, University of Calgary
Differential Categories to Tangential Structure

In this talk I describe work with Goeff Cruttwell and Johnathan Gallagher on differential categories and, in particular, on the passage from differential to tangential restriction categories.

The study of abstract differential structure for categories was initiated by Thomas Ehrhard who discovered a number of models of differential structures arising from his work in linear logic. These are interesting models which have computational content which appears to be related to the semantics of distributed systems.

This work exposed the purely algebraic structure underlying differentiation. In 2008, together with Robert Seely and Rick Blute, we introduced Cartesian differential categories. An important way in which these arise is as the coKleisli category of a models of differential linear logic (as mentioned above). However, significantly, they also arise much more directly from standard (algebraic and synthetic) models of the ordinary differential calculus (on the real line).

Given these of purely algebraic/categorical descriptions of differentiation, the possibility of capturing abstractly some of the basic ideas of differential geometry (e.g. smooth manifolds etc.) came within reach. To this end we introduced the notion of a differential restriction category: this adds partiality (and therefore topology) to the differential structure. From there one can use the manifold construction of Marco Grandis to obtain categories of "smooth manifolds". Furthermore, one can axiomatize the structure which arises using tangent spaces – here called tangential structure.

An important example of all this structure, which I will discuss, is central to algebraic geometry.

MARTIN FRANKLAND, University of Illinois at Urbana-Champaign

Moduli spaces of 2-stage Postnikov systems

It is a classic fact that any graded group (abelian above dimension 1) can be realized as the homotopy groups of a space. However, the question becomes difficult if one includes the data of primary homotopy operations, known as a Π-algebra. When a Π-algebra is realizable, we would also like to classify all homotopy types that realize it.

Using an obstruction theory of Blanc-Dwyer-Goerss, we will describe the moduli space of realizations of certain 2-stage Π-algebras. This is better than a classification: The moduli space provides information about realizations as well as their higher automorphisms.

RICK JARDINE, University of Western Ontario

Dynamical systems and diagrams

A dynamical system is a space X with a pairing from $X \times S \to X$ for some parameter space S, and a map of such dynamical systems is an S-equivariant map. There is an injective and a projective model structure for the resulting category of spaces with S-action, and both are easily derived.

These model structures are special cases of model structures for presheaf-valued diagrams X defined on a fixed presheaf of categories E which is enriched in simplicial sets.

Simultaneously varying the parameter category object E (or parameter space S) along with the diagrams X up to weak equivalence is more interesting, and requires new model structures for E-diagrams having weak equivalences defined by homotopy colimits, as well as a generalization of Thomason's model structure for small categories to a model structure for presheaves of simplicial categories. These new model structures exist for arbitrary presheaves of simplicial categories E and their categories of diagrams.

GÁBOR LUKÁCS, University of Manitoba

On abelian topobornological groups: A convenient category for duality theory

A *group bornology* on a group G is a bornology that makes the group operations bounded. An abelian *topobornological group* consists of an abelian group G, a group topology on G, and a group bornology on G. The category of abelian topobornological groups and continuous bounded homomorphisms has a well-behaved internal hom-functor, which makes it suitable for studying duality theories of topological groups, and lends itself to a unified duality theory that includes the Pontryagin duality and the Comfort-Ross duality.

PETER LEFANU LUMSDAINE, Dalhousie University

"Axiomatising the circle": higher inductive types in dependent type theory

The emerging field of "homotopy type theory" investigates logical systems whose primitive objects behave not like sets, but more like homotopy types.

In type theory, many important objects are defined inductively: for instance, the natural numbers are defined as the type freely generated by an element '0' and an endofunction 'suc'. This gives the principles of induction and recursion which characterise $\mathbb{N}$. More general "inductive type definitions" form the most important construction principle in various type theories.

What if one similarly defines the circle to be the type freely generated by a single point and a path from that point to itself? This gives induction/recursion principles for the circle; it turns out many facts of classical algebraic topology are directly provable in the type theory using this definition.

More generally, adding these "higher inductive types", which may be freely generated not only by points but also by paths, homotopies, and so on, gives a very powerful construction principle. Using these, one can reconstruct many topological constructions type-theoretically: spheres, more general cell complexes, mapping cylinders, truncations, and much more.

MICHAEL MAKKAI, McGill University
Generalization of FOLDS (First Order Logic with Dependent Sorts) in a hyperdoctrinal setting

FOLDS was introduced in M. Makkai: First Order Logic with Dependent Sorts, with Application to Category Theory (1995; www.math.mcgill.ca/makkai/), cited below as "FOLDS I", and M. Makkai: Towards a Categorical Foundation of Mathematics, in: Logic Colloquium '95, Lecture Notes in Logic 11, 1998; pp. 153-190. In an unpublished manuscript from 1998, henceforth cited as "FOLDS II", the theory is reworked and generalized. In the talk, I will give a description of some of the contents of FOLDS II, as well as present further related results. The main feature of FOLDS is the concept of "FOLDS equivalence". With any FOLDS signature L (a skeletal one-way category with finite fan-out), there is associated, among others, a relation on L-structures, called "L-equivalence". First-order statements on L-structures that are invariant under L-equivalence are, up to logical equivalence, precisely the ones that are formulated in the syntax of FOLDS (General Invariance Theorem (GIT); see FOLDS I). FOLDS II and the newer work contain a full treatment of the GIT, in the generalized context, for both classical and intuitionistic logic as well as infinitary extensions of first order logic. FOLDS II is formulated in a purely categorical language, in the spirit of M. Makkai: The Fibrational Formulation of Intuitionistic Predicate Logic, Notre Dame J. Formal Logic 1993, pp. 334-377 and 471-498. In FOLDS II, a logical theory is taken to be a "Q-fibration". In Appendix B of FOLDS I, the concept of Q-fibration is briefly discussed.

BOB PARÉ, Dalhousie University
The Double Category of Lax Presheaves

A study of representables for double categories shows that they are lax functors into a double category of sets whose vertical arrows are spans. This leads to a Yoneda embedding into a double category of lax presheaves with horizontal morphisms called natural transformations (generalizing lax transformations) and vertical morphisms called modules (generalizing profunctors). This double category is the foundation upon which two-dimensional model theory will be built, and it is important to establish its basic properties. The most urgent questions surround the composition of modules. Does it exist? Is it associative? Etc. In the course of discussing these questions, several double category constructions of independent interest will arise.

EMILY RIEHL, University of Chicago
Algebraic model structures

Algebraic model structures are an extension of Quillen's model categories in which the functorial factorizations define monads and comonads. In the presence of this structure, the (co)fibrations can be regarded as (co)algebras for the (co)monad. The (co)algebra structures witness the fact that a particular map is a (co)fibration and can be used to construct a canonical solution to any lifting problem. For example, the algebraic structure for Hurewicz fibrations is a path lifting function; for Kan fibrations it is a choice of fillers for all horns. Despite this rigid structure, which in particular includes a (co)fibrant replacement (co)monad, algebraic model structures exist for most cofibrantly generated model categories. We describe a few features of this theory and then define and characterize algebraic Quillen adjunctions, in which the functors must preserve algebraic (co)fibrations, not simply ordinary ones. We conclude with a brief discussion of new work defining and characterizing monoidal and enriched algebraic model structures that gives particular emphasis to the role played by "cellularity" of certain cofibrations.

LAURA SCULL, Fort Lewis College
Simplicial Structures in 2-Categories

I will discuss an ongoing project to produce and understand the correct 2-categorical version of the Δ simplicial category. Our approach is through monoids: MacLane has shown that the usual simplicial Δ category is the universal monoid, in the sense that there is a unique morphism from Δ to any other monoid. The goal of this project is to produce a 2-Δ category which will have an analogous property to this. This is work in progress, with no conclusion at this time, but I will be discussing some of the considerations in making this work. This is joint work with John MacDonald (UBC).

JEFF SMITH, UBC

DON STANLEY, Regina

CLAIRE TOMESCH, University of Chicago
Segalic Models of (∞, n)-categories

Simpson and Tamsamani have both given definitions of weak n-category involving simplicial objects in a given category which satisfy the Segal condition and whose zeroeth level ('object of objects') is, in an appropriate sense, discrete. We give a general framework for studying Segalic models of (∞, n)-categories – explicitly, models that rely on a notion of discrete object.

MICHAEL WARREN, Dalhousie University
Recent advances relating homotopy theory and type theory

It has been recently observed that there are number of interesting connections between homotopy theory and Martin-Löf type theory. In this talk we will survey of a number of recent advances relating these areas. In particular, we will describe several aspects of Voevodsky's *univalent foundations of mathematics* and some recent results by the speaker and others in connection with this project.