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"Axiomatising the circle": higher inductive types in dependent type theory

The emerging field of "homotopy type theory" investigates logical systems whose primitive objects behave not like sets, but more like homotopy types.

In type theory, many important objects are defined inductively: for instance, the natural numbers are defined as the type freely generated by an element '0' and an endofunction 'suc'. This gives the principles of induction and recursion which characterise $\mathbb{N}$. More general "inductive type definitions" form the most important construction principle in various type theories.

What if one similarly defines the circle to be the type freely generated by a single point and a path from that point to itself? This gives induction/recursion principles for the circle; it turns out many facts of classical algebraic topology are directly provable in the type theory using this definition.

More generally, adding these "higher inductive types", which may be freely generated not only by points but also by paths, homotopies, and so on, gives a very powerful construction principle. Using these, one can reconstruct many topological constructions type-theoretically: spheres, more general cell complexes, mapping cylinders, truncations, and much more.