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A comparative study of extrapolation methods, sequence transformations and steepest descent methods for semi-infinite integrals

With the advent of computers and scientific computing, there has been a push to develop more accurate, more efficient, and more reliable techniques in computing challenging problems in applied mathematics. In the numerical evaluation of infinite-range integrals, a common problem in applied mathematics, three general methods have come to the forefront, and have done so largely through independent trains of thought. To wit, these methods are known as extrapolation methods, sequence transformations and steepest descent methods.

In extrapolation methods, through numerical quadrature or otherwise, one computes a sequence of approximations to the infinite-range integral and uses analytical properties of the integrand to then extrapolate on this sequence to obtain an approximation for the integral. In sequence transformations, one derives the asymptotic series expansion of the integral and, whether convergent or divergent, one applies transformations to the asymptotic series hoping to approximate the limit or antilimit of the series with a relatively small number of terms. In the steepest descent methods, a deformation of the path of integration is used to transform oscillations or irregular exponential behaviour into linear exponential decay. On the deformed contour, a Gauss-Laguerre-type quadrature is used to approximate the integral.

In this work, we put these three general methods to the test on five prototypical infinite-range integrals exhibiting oscillatory, logarithmic and exponential properties or combinations thereof. On the bases of accuracy, efficiency, simplicity, and reliability, we compare and contrast the three general methods for the evaluation of infinite-range integrals.