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A Problem of Fomenko's related to Artin's Conjecture on Primitive Roots

Let a be a fixed, squarefree integer. In 1927, Artin conjectured that a is a primitive root modulo p for a positive proportion of primes up to x . Hooley has shown that this is true upon the GRH. We view this conjecture in a different light and consider a generalization. In particular, let $i_a(p) := [(\mathbb{Z}/p\mathbb{Z})^* : \langle a \bmod p \rangle]$. Then, we consider summations of the form

$$\sum_{p \leq x} f(i_a(p))$$

where $f : \mathbb{N} \rightarrow \mathbb{C}$ is a function. We will see that for a large class of functions, the above summation satisfies

$$\sum_{p \leq x} (\log(i_a(p)))^\alpha = c_a \pi(x) + O(x/(\log x)^{2-\varepsilon-\alpha})$$

for any $\alpha \in (0, 1)$ and $\varepsilon > 0$, and where c_a is a constant dependent on a , and

$$\sum_{p \leq x} \tau(i_a(p)) = c'_a \pi(x) + O(x/(\log x)^{2-\varepsilon})$$

for any $\varepsilon > 0$ and where c'_a is a constant dependent on a .