LOUIS DEAETT, University of Victoria

The principal rank characteristic sequence of a real symmetric matrix

Given an $n \times n$ real symmetric matrix A we associate to A a sequence $r_0 r_1 \cdots r_n \in \{0,1\}^{n+1}$ defined by

$$r_k = \begin{cases} 1 & \text{if } A \text{ has a principal submatrix of rank } k \text{, and} \\ 0 & \text{otherwise,} \end{cases}$$

or, equivalently,

$$r_k = \begin{cases} 1 & \text{if } A \text{ has a nonzero principal minor of order } k \text{, and} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

for $1 \leq k \leq n$, with $r_0 = 1$ if and only if A has a zero entry on its main diagonal. Denote this sequence by $\mathrm{pr}(A)$.

Now, given an arbitrary sequence of 0s and 1s, is it $\mathrm{pr}(A)$ for any real symmetric matrix A? If so, call the sequence $attainable$. The problem, then, is to characterize the attainable sequences.

We will discuss how this problem relates to graph eigenvalues and to both some quite old and some quite recent results concerning algebraic relationships between the principal minors of a symmetric matrix.

Joint work with Richard Brualdi, Dale Olesky and Pauline van den Driessche.