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*C<sup>1</sup> fine approximation on Banach spaces*

We consider the so-called  $C^1$  fine approximation on Banach spaces. Specifically, given Banach spaces  $X$  and  $Y$ , a  $C^1$  map  $f : X \rightarrow Y$ , and  $\varepsilon : X \rightarrow (0, \infty)$  a continuous function, for  $p \in (1, \infty] \cup \omega$  can we find a  $C^p$  smooth function  $g : X \rightarrow Y$  so that

$$\|f(x) - g(x)\| < \varepsilon(x),$$

and

$$\|f'(x) - g'(x)\| < \varepsilon(x).$$

In general this turns out to be a surprisingly difficult problem which is closely related to the ability to approximate Lipschitz maps via Lipschitz, smooth maps with good control over the Lipschitz constant.

We discuss some classical and recent results.