CHRIS LENNARD, University of Pittsburgh

The closed, convex hull of every ai c_0-summing basic sequence fails the FPP for affine nonexpansive mappings.

In 2004 Dowling, Lennard and Turett showed that every non-weakly compact, closed, bounded, convex (c.b.c.) subset K of $(c_0, \|\cdot\|_\infty)$ is such that there exists a $\|\cdot\|_\infty$-nonexpansive mapping T on K that is fixed point free. This mapping T is generally not affine. It is an open question as to whether or not on every non-weakly compact, c.b.c. subset K of $(c_0, \|\cdot\|_\infty)$ there exists an affine $\|\cdot\|_\infty$-nonexpansive mapping S that is fixed point free.

In a recently accepted joint paper with Veysel Nezir, we prove that if a Banach space contains an asymptotically isometric (ai) c_0-summing basic sequence $(x_n)_{n\in\mathbb{N}}$, then the closed convex hull of $(x_n)_{n\in\mathbb{N}}$, $E := \overline{co}(\{x_n : n \in \mathbb{N}\})$, fails the fixed point property for affine nonexpansive mappings. Moreover, we show that there exists an affine contractive mapping $U : E \longrightarrow E$ that is fixed point free. Furthermore, we prove that for all sequences $\overrightarrow{b} = (b_n)_{n\in\mathbb{N}}$ in $\mathbb{R}$ with $0 < m := \inf_{n\in\mathbb{N}} b_n$ and $M := \sup_{n\in\mathbb{N}} b_n < \infty$, the closed, bounded, convex subset $E = E_{\overrightarrow{b}}$ of c_0 defined by

$$E := \left\{ \sum_{n=1}^{\infty} t_n \, f_n : 1 = t_1 \geq t_2 \geq \cdots \geq t_n \downarrow_n 0 \right\} ,$$

where each $f_n := b_n \, e_n$, is such that there exists an affine contractive mapping $U : E \longrightarrow E$ that is fixed point free.