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The Matrix Extension Problem and Orthogonal Wavelets over Algebraic Number Fields

As a finite dimensional linear space over the rational number field, an algebraic number field is of particular interest in mathematics and engineering. Algorithms using algebraic number fields can be efficiently implemented involving only integer arithmetics. We observe that all finitely supported orthogonal low-pass (multi)wavelet filters known in the literature have coefficients from an algebraic number field. Therefore, it is of interest to study orthogonal (multi)wavelet filter banks over algebraic number fields. In this talk, we shall formulate the matrix extension problem over a general subfield of the complex number field (including algebraic number fields as special cases). The core task of the matrix extension problem is to extend a given $r \times s$ matrix, with $1 \le r \le s$, of Laurent polynomials into an $s \times s$ square paraunitary matrix with some desirable properties such as symmetry and short supports. We shall provide a complete satisfactory solution to this matrix extension problem and discuss its applications to symmetric orthogonal (multi)wavelets. Several examples of symmetric real-valued or complex-valued orthogonal wavelets are provided to illustrate the results. This is joint work with Xiaosheng Zhuang. Preprints are available at http://www.ualberta.ca/$\sim$bhan