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Faà di Bruno Categories

In several papers, Blute, Cockett and Seely have described a categorical approach to differential calculus based on intuitions (due to T. Ehrhard) from linear logic. Specifically, they presented a comonadic setting, whose functor is a differential operator, whose base maps are linear and whose coKleisli maps are smooth. There are two complementary approaches to differential categories: differential categories (where the emphasis is on the base category with a differential comonad) and Cartesian differential categories (where the emphasis is reversed, presenting the category of smooth maps inside which lives the category of linear maps). There are natural connections between these two notions, but they are not equivalent (unless considerably strengthened).

In the Cartesian differential categorical context, the structure of the chain rule gives rise to a fibration, the "bundle category". In this talk I shall generalise this to the higher order chain rule (originally developed in the traditional setting by Faà di Bruno in the nineteenth century); given any Cartesian differential category X, there is a "higher-order chain rule fibration" $\mathrm{Faa}(X) \to X$ over it. In fact, Faa is a comonad (over the category of Cartesian left semi-additive categories); the main theorem is that the coalgebras for this comonad are precisely the Cartesian differential categories.

Joint work with Robin Cockett.