JUAN CARLOS BUSTAMANTE, Sherbrooke

Special biserial algebras with no outer derivations

Let k be a field. Given a special biserial $A = kQ/I$, we investigate the relationships between its simple connectedness, the vanishing of its Hochschild cohomology groups, and some combinatorial data for (Q, I).

A bound quiver (Q, I) is said to be special biserial if it satisfies the following:

(i) every vertex in Q is the source of at most two arrows and the target of at most two arrows;

(ii) given an arrow $\alpha\colon x \to y$ in Q, there is at most one arrow β starting from y such that $\alpha\beta \notin I$ and there is at most one arrow γ arriving at x such that $\gamma\alpha \notin I$.

Consider the special biserial algebra $A = kQ/I$, and let S be the two-sided ideal of A generated by classes of paths appearing in binomial relations, so that A/S is a string algebra.

Moreover, denote by $\mathrm{HH}^i(A) := \mathrm{Ext}^i_{A-A}(A, A)$ the Hochschild cohomology groups of A with coefficients in the bimodule ${}_AA_A$. The sum $\mathrm{HH}^*(A) := \bigoplus_{i \geqslant 0} \mathrm{HH}^i(A)$ is then a graded commutative ring for the Yoneda product and a graded Lie algebra for a bracket defined using composition of cochains.

An algebra A is said to be simply connected if it has no proper Galois cover. This amounts to say that if $A \simeq kQ/I$ then the fundamental group $\pi_1(Q, I)$ is trivial.

The main result of this work is:

Theorem *Let $A = kQ/I$ be a special biserial algebra. Then the following conditions are equivalent.*

(1) A is simply connected of finite representation type;

(2) $\mathrm{HH}^1(A) = 0$;

(3) $\mathrm{HH}^*(A) = k$;

(4) $\dim_k S = \chi(Q)$.

We shall discuss about the proof and some consequences.

This is a joint work with Ibrahim Assem, and Patrick Le Meur.