Mathematical Perspectives on Quantum Theory and Gravity
Perspectives mathématiques sur théorie quantique et gravitation
(Org: **Jack Gegenberg** and/et **Viqar Husain** (UNB))

GLENN BARNICH, Université Libre de Bruxelles & International Solvay Institutes
Asymptotically flat spacetimes revisited: aspects of the BMS/CFT correspondence

After a brief review of purely gravitational aspects of the AdS3/CFT2 correspondence, a similar analysis is performed for asymptotically flat spacetimes at null infinity in 3 and 4 dimensions. In the spirit of two dimensional conformal field theory, it is shown that the symmetry algebra of asymptotically flat spacetimes at null infinity in 4 dimensions can be taken to be the semi-direct sum of supertranslations with infinitesimal local conformal transformations and not, as usually done, with the Lorentz algebra. As a first application, we derive how the symmetry algebra is realized on solution space. In particular, we work out the behavior of Bondi's news tensor, mass and angular momentum aspects under local conformal transformations. We comment on the implications for the problem of angular momentum in general relativity.

JULIANE BEHREND, St. Francis Xavier University, Antigonish, NS, B2G 2W5
Covariant Averaging in a Class of Metric-Affine Theories of Gravity

The averaging problem is a longstanding unresolved problem of general relativity. One of the concerns in the averaging problem is the question of how to define an average over tensor quantities in a physically sensible manner when the tensors are defined at separate spacetime points. In our novel approach to the problem we make use of an equivalence between general relativity and a class of teleparallel theories of gravity. When a teleparallel structure exists on a manifold a covariant averaging process can be formulated easily with the framework of metric-affine theories of gravity. This averaging procedure will be illustrated with a specific example and cosmological implications will be discussed.

IVAN BOOTH, Memorial University, St. John's, Newfoundland and Labrador
Black brane entropy and hydrodynamics

For stationary black holes it is universally known that entropy is proportional to horizon area. It is not so clear what the relationship is for dynamical black holes. In such spacetimes the event horizon is teleologically defined while the apparent horizon is non-unique. Thus even if one believes that entropy continues to be well-defined and proportional to horizon area, there are many possible areas to choose from. In this talk I will review some recent work done in collaboration with MichałHeller and MichałSpaliński that examines this problem in the context of dynamical black branes in AdS spacetimes. We work in the quasi-equilibrium regime where the horizons are slowly evolving on the gravity side while the field theory side can be described by hydrodynamics. Then a hydrodynamic entropy current can be defined and compared with the various horizon areas. We examine and attempt to identify the uncertainties in each.

JOHAN BRANNLUND, Dalhousie University, Halifax, NS B3H 3J5
Covariant tensor averaging

When applying general relativity to cosmology, one may take the point of view that one should start with a "lumpy" spacetime and average this to smooth out inhomogeneities, as opposed to postulating an FRW model from the outset. Various averaging procedures have previously been proposed but suffer from some problems, such as a lack of general covariance.

Here, a covariant procedure for averaging tensor fields on a manifold will be presented. Examples, applications and limitations will be discussed.

Based on joint work with Robert van den Hoogen and Alan Coley.

GOLAM HOSSAIN, University of New Brunswick
Scalar field theory and polymer quantization

To quantize gravity sector in loop quantum gravity approach, one uses the so-called polymer quantization rather than the usual Fock quantization. For completeness, such a quantization should also be applied to the matter sector. In this talk, I discuss some consequences of polymer quantization in scalar field theory.

ANDREAS KREIENBUEHL, UNB Fredericton
Ultraviolet Behavior in Background Independent Quantum Field Theory

We describe a background independent quantization of the scalar field that provides an explicit realization of Fock-like states and associated operators in a polymer Hilbert space. The vacuum expectation values of the commutator and anticommutator of the creation and annihilation operators become energy dependent, and exhibit a surprising transition to fermionic behavior at high energy.

Furthermore, the approach yields a modified dispersion relation with a leading correction proportional to the momentum cubed. These results suggests a fundamental change in the ultraviolet properties of quantum fields.

HARI KUNDURI, Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, AB T6G 2J1
An infinite class of extremal horizons in higher dimensions

The classification problem of stationary black holes in higher dimensions is a challenging open problem of intrinsic interest, and is furthermore relevant to high-energy physics. It has been established that allowed horizon topologies must be oriented cobordant to a sphere and, at least in the asymptotically flat case, be of positive Yamabe type. These conditions are only necessary; however, by restricting attention to extremal black holes, one can analyze the field equations in detail and impose regularity.

I will present a new class of near-horizon geometries which solve Einstein's vacuum equations, including a negative cosmological constant, in all even dimensions greater than four. Spatial sections of the horizon are inhomogeneous S^2-bundles over any compact Kaehler–Einstein manifold. For a given base, the solutions are parameterised by one continuous parameter (the angular momentum) and an integer which determines the topology of the horizon. In six dimensions the horizon topology is either $S^2 \times S^2$ or $CP^2 \# \text{-}CP^2$. In higher dimensions the S^2-bundles are always non-trivial, and for a fixed base, give an infinite number of distinct horizon topologies. All of these horizon geometries are consistent with all known topology and symmetry constraints for the horizons of asymptotically flat or globally anti-de Sitter extremal black holes.

GABOR KUNSTATTER, University of Winnipeg
Quantum Corrected Spherical Collapse: A Phenomenological Framework

A phenomenological framework is presented for incorporating quantum gravity motivated corrections into the dynamics of spherically symmetric collapse. The effective equations are derived from a variational principle that guarantees energy conservation and the existence of a Birkhoff theorem. The gravitational potential can be chosen as a function of the areal radius to yield specific non-singular static spherically symmetric solutions that generically have two horizons. For a specific choice of potential the effective stress energy tensor violates only the dominant energy condition. The violations are maximum near the inner horizon and die off rapidly. A numerical study of the quantum corrected collapse of a spherically symmetric scalar field reveals that the modified gravitational potential prevents the formation of a central singularity and ultimately yields a static, mostly vacuum, spacetime with two horizons. The matter "piles up" on the inner horizon giving rise to mass inflation at late (Vaidya) times. The Cauchy horizon is transformed into a null, weak singularity, but in contrast to Einstein gravity, the absence of a central singularity renders this null singularity stable.

Joint work with J. Ziprick, University of Waterloo and Perimeter Institute.

TOMAS LIKO, Pennsylvania State University
A unified treatment of asymptotically flat and anti-de Sitter spacetimes

The action for gravity with an asymptotic boundary must be renormalized; in the case of asymptotically flat spacetimes the action itself is infinite when evaluated for Minkowski spacetime, while for asymptotically anti-de Sitter spacetimes the boundary stress tensor on the conformal boundary is infinite. For these spacetimes, different prescriptions for generating infinite counter-terms are typically used in order to obtain finite quantities. A new prescription is discussed here which suggests that there exists a *generic counter-term* for which both the action on asymptotically flat spacetimes and the boundary stress tensor on asymptotically anti-de Sitter spacetimes are rendered finite. This counter-term arises from the first-order action on the configuration space consisting of the coframe and (Lorentz) connection; the corresponding boundary term is the natural one that appears from the requirement of functional differentiability. Extension to supergravity is also briefly discussed.

ROBERT MANN, Dept. of Physics & Astronomy, University of Waterloo
Lifshitz Black Holes

Lifshitz black holes are black hole solutions whose asymptotic spacetime structure yields a boundary theory that has been conjectured to be dual to a Lifshitz theory describing critical phenomena in one less dimension. I describe a set of black hole solutions that are expected to be dual to a Lifshitz fixed point geometry and depend on a single parameter that determines both their area (or size) and their charge. Most of the solutions are obtained numerically, but an exact solution is also obtained for a particular value of this parameter. I also discuss the effects that higher-order curvature corrections have on these solutions. The thermodynamic behaviour of large black holes is almost the same regardless of genus, but differs considerably for small black holes.

ROBERT MILSON, Dalhousie University
The type N Karlhede bound is sharp

We present an example of a family of four-dimensional Lorentzian manifolds whose invariant classification requires the seventh covariant derivative of the curvature tensor. The spacetimes in questions are null radiation, type N solutions on an anti-de Sitter background. The upper bound of 7 on the order of the covariant derivative, was first established by Karlhede, about 30 years ago. Thanks to this result, we now know that this bound is sharp. The key technique for the search of such anomalous spacetimes is the property of curvature homogeneity, meaning that the tetrad components of the Riemann curvature tensor and its covariant derivatives up to a certain order can be taken to be constant.

DAVID RIDEOUT, Perimeter Institute for Theoretical Physics
Oriented Matroids as Combinatorial Structures Underlying Loop Quantum Gravity

We analyze combinatorial structures which play a central role in determining spectral properties of the volume operator in loop quantum gravity (LQG). These structures encode geometrical information of the embedding of arbitrary valence vertices of a graph in 3-dimensional Riemannian space, and can be represented by sign strings containing relative orientations of embedded edges. We demonstrate that these signature factors are a special representation of the general mathematical concept of an oriented matroid. Moreover, we show that oriented matroids can also be used to describe the topology (connectedness) of directed graphs. Hence the mathematical methods developed for oriented matroids can be applied to the difficult combinatorics of embedded graphs underlying the construction of LQG. As a first application we revisit our earlier analysis of the volume spectrum, and find that the enumeration of all possible sign configurations used there is equivalent to enumerating all realizable oriented matroids of rank 3, and thus can be greatly simplified. We find that for 7-valent vertices having no coplanar triples of edge tangents, the smallest non-zero eigenvalue of the volume spectrum does not grow as one increases the maximum spin jmax at the vertex, for any orientation of the edge tangents. This indicates that, in contrast to the area operator, considering large jmax does not necessarily imply large volume eigenvalues. In addition we give an outlook to possible starting points for rewriting the combinatorics of LQG in terms of oriented matroids.

KRISTEN SCHLEICH, University of British Columbia
Trapped Surfaces and Seiberg–Witten invariants

We prove existence of trapped surfaces in a certain class of asymptotically flat initial data sets of 3 or more dimensions. This class is characterized by a new smooth invariant, the maximal Yamabe invariant, defined by smooth compactification of the asymptotically flat manifold. These results apply to spacetimes admitting a Cauchy surface of nonpositive maximal Yamabe invariant with initial data that satisfies the dominant energy condition. For 4-dimensional asymptotically flat initial data, the sign of the maximal Yamabe invariant can be related to the Seiberg–Witten invariants of the smooth compactification. In particular, 5-dimensional spacetimes with smooth Cauchy surfaces with non-trivial Seiberg–Witten invariants must contain a trapped surface.

SANJEEV SEAHRA, University of New Brunswick
Implications of polymer quanization in cosmology

I discuss the implications of polymer quantization of scalar matter for early universe cosmology. In particular, we discuss how a particular polymer quantization scheme can lead to a de Sitter-like inflationary epoch at early times.

DON WITT, University of British Columbia
Soap Bubbles and $2+1$-Gravity

The existence of marginally outer trapped surfaces in $2+1$ gravity is mathematically very similar to the Plateau problem. This classic problem is to show the existence of a minimal surface satisfying a prescribed boundary condition. Soap bubbles suspended on a wire frame are a physical realization of these mathematical results. The existence of marginally outer trapped surfaces in $2+1$ gravity is determined by similar equations and boundary conditions. However, the outcome for this case is completely different, yielding non-existence results for marginally outer trapped surfaces in $2+1$ gravity. These results also apply in solution space quantizations of $2+1$-dimensional gravity.

ERIC WOOLGAR, University of Alberta
Ricci flow of the RP3 geon, and beyond

The Ricci flow has proved to be a powerful tool in Riemannian geometry, at least on compact manifolds. Application to problems in general relativity requires dealing with several issues, including the presence of boundaries and asymptotic regions and (sometimes) the signature of the metric. For example, the presence of an asymptotically flat region modifies the standard proof that the Ricci flow of a compact 3-manifold with essential minimal surface present must terminate in finite time, leaving open (for now at least) the possibility that, for certain initial data, this flow could be immortal. I will discuss this result (including the Ricci flow of the RP3-geon model) and provide results of a numerical investigation. Time permitting, I will also pose some questions that arise from efforts to apply Ricci flow to general relativity and string theory.

Based on joint work with Tracey Balehowsky.