Noncommutative Geometry
Géométrie non commutative
(Org: **Bahram Rangipour** (UNB))

PAUL BAUM, Pennsylvania State University, University Park, PA 16802, USA
Fine structure in the K-theory of reductive p-adic groups

Let G be a reductive p-adic group. Examples are $\mathrm{GL}(n, Q_p), \mathrm{SL}(n, Q_p)$, etc., where Q_p is the field of p-adic numbers. Since Q_p is a locally compact topological field, these groups are locally compact. If G is any reductive p-adic group, V. Lafforgue has proved that the BC (Baum–Connes) conjecture is valid for G. However, there is a fine structure in the K-theory of $C *_r (G)$ which BC does not seem to explicitly calculate. This talk will explain what this fine structure is and will then state a conjecture as to how this fine structure can be explicitly calculated. This fine structure conjecture then leads to the ABP (Aubert–Baum–Plymen) conjecture in the representation theory of reductive p-adic groups.

MARGARET BEATTIE, Mount Allison University
Some results for dual quasi-Hopf algebras

Just as noncommutative, noncocommutative Hopf algebras provide a natural setting for quantum group theory, so generalizations of Hopf algebras may provide a suitable setting for dynamical quantum group theory. This talk will describe two examples of encounters with dual quasi-Hopf algebras and the difficulties and differences from the classical theory.

GEORGE ELLIOTT, University of Toronto
An abstract characterization of the index of a Fredholm operator

The universal enveloping abelian semigroup of the semigroup of Fredholm operators (on an infinite-dimensional Hilbert space) is the group of integers. A similar statement holds (with a similar proof) for the semigroup of Fredholm elements of the multiplier C^*-algebra of an arbitrary stable C^*-algebra (provided that this is assumed to have an approximate unit consisting of projections—presumably just a technical hypothesis). Of course, the group of integers is replaced by a different group in the general case.

This is joint work with Brian Skinner.

FARZAD FATHIZADEH, University of Western Ontario, 1151 Richmond St., London, ON N6A 5B7
The Gauss–Bonnet theorem for the noncommutative torus II

We shall first give an outline of a spectral formulation and proof of the Gauss–Bonnet theorem for the noncommutative two torus due to Connes and Tretkoff. Then we shall explain our ongoing work in progress where this result is generalized to arbitrary values of the complex structure. This involves some heavy computer calculations with the heat kernel.

This is joint work with M. Khalkhali.

ALEXANDER GOROKHOVSKY, University of Colorado at Boulder
Local index theorem for projective families

Mathai, Melrose and Singer stated and proved an index theorem for twisted families of elliptic operators. In this talk I will describe a superconnection proof of this theorem (with some conditions relaxed).

This is a joint work with M. Benameur.

PIOTR M. HAJAC, IMPAN / Warsaw University, Warsaw, Poland
A new type of cocyclic modules

For finite-dimensional Hopf algebras, anti-Yetter–Drinfeld modules are equivalent to modules over the anti-Yetter–Drinfeld algebra (a Galois object over the Drinfeld double). We propose a definition of the anti-Yetter–Drinfeld algebra that works for infinite-dimensional Hopf algebras. Again, each anti-Yetter–Drinfeld module is a module over such an algebra, but now there more modules over this algebra than anti-Yetter–Drinfeld modules. For the thus constructed more general type of modules, we define a cocyclic complex with coefficients in such a module.

Joint work with Gabriella Bohm.

MOHAMMAD HASSANZADEH, University of Western Ontario, Middlesex College, London, Ontario N6A 5B7
Cup products in Hopf cyclic cohomology

We define coproducts for Hopf cyclic cohomology of cocommutative Hopf algebras. In the periodic case, we show that this coproduct agrees with the coproduct of Lie algebra homology. We also introduce coproducts for dual Hopf cyclic cohomology of cocommutative Hopf algebras. At the end we introduce a graded commutative and associative algebra structure on dual Hopf cyclic cohomology of commutative Hopf algebras.

This is joint work with Masoud Khalkhali.

MAX KAROUBI, Université Paris 7
Bott periodicity for discrete orthogonal groups

Bott periodicity is one of the fundamental theorems in K-theory. We revisit the subject, after the fundamental work of Suslin, Voevodsky and Rost. We prove that the homotopy groups with finite coefficients of the "discrete" infinite orthogonal group are also periodic, after a certain range depending on the étale codimension of the ring involved.

This is a joint work with Jon Berrick and Paul Ostvaer.

ULRICH KRAEHMER, University of Glasgow, Department of Mathematics, University Gardens, Glasgow, Scotland G12 8QW
On the NCG of the Podles sphere (again)

I'll report on joint work with Elmar Wagner concerning the noncommutative geometry of the Podles quantum sphere. This is work in progress, but in any case I will present the fundamental Hochshcild (co)homology class in terms of spectral triples.

TOMASZ MASZCZYK, Institute of Mathematics, Polish Academy of Sciences, and University of Warsaw, Warsaw, Poland
Noncommutative algebraic sets

We construct a strong monoidal faithful and full embedding of the cartesian monoidal category of sets into 1-cells of some 2-category. These 1-cells are opmonoidal functors (admitting the right adjoint) between some monoidal categories ("quantum vector spaces over quantum fields"). The image of this embedding will be regarded as picking classical sets among their quantum counterpart. Next, I will show how to endow (convolution) representations of "quantum fields" (some associative unital algebras) with a structure of an enriched category, whose morphisms form "quantum sets" as above. This category should be understood as "quantum category of quantum fields". It generalizes its classical counterpart, in particular the classical Galois group. For any associative algebra we construct an enriched functor ("functor of quantum points") from the enriched category of "quantum fields" to the category of "quantum sets".

ARASH POURKIA, The University of Western Ontario
A super version of the Connes–Moscovici Hopf algebra, $\mathcal{H}_1$

We define a super version of the Connes–Moscovici Hopf algebra, $\mathcal{H}_1$. For that, we define the super group $G^s = \mathrm{Diff}^+(\mathbb{R}^{1,1})$ of the orientation preserving diffeomorphisms of the super line $\mathbb{R}^{1,1}$ and introduce two (super) sub-groups G_1^s and G_2^s of G^s, where G_1^s is the super group of affine transformations. The super Hopf algebra $\mathcal{H}_1^s$ is defined as a certain bicrossproduct super Hopf algebra. This bicrossproduct is constructed by two super Hopf algebras attached to G_1^s and G_2^s.

GEORGY SHARIGYN, ITEP, Moscow
Hopf-cyclic cohomology and pairings: examples

In my talk I will give few explicit examples of pairings between the Hopf-cyclic cohomology of algebras and coalgebras in case when both ingredients arise from geometric constructions. Our examples will include universal enveloping algebras of vector fields and group algebras acting on functions on manifolds, bar-resolution of an algebra, coacting on the algebra of functions on a principal bundle etc. In all these cases we shall express the pairing in question in more classical (geometric) terms.

REZA G. SHOUSHTARI, UNB, No. 33–31(C), Waggoner's Lane, Fredericton, NB
Hopf–Galois coextension and SAYD modules

Cyclic (co)homology theory as any other cohomology theory is based on (co)chain together with (co)boundary operator. The cochain that we consider are simplicial cochains which are also cyclic. In some interesting case we can define a (co)simplicial structure over a module. The most natural simplicial structure happens in geometry. One also associates simplicial structure to tensor algebra of any algebra or coalgebra. The result is cyclic module associated to the algebra or the coalgebra. Now one considers an ordinary space and makes an algebra out of that space by means of smooth functions. The beautiful theorem of Alain Connes says that the deRham cohmology of the space in question and the cyclic cohomology of the algebra of smooth function on the space coincide. We now study a situation a bit more interesting by letting the coefficients also play some role. The new coefficients here are called SAYD modules. The new theory in addition to an algebra or coalgebra needs a source of symmetry which is usually provided by an action of a Hopf algebra. Stable Anti Yetter Driffield (SAYD) modules are special type of a module and comodule over a Hopf algebra with AYD and Stability conditions. By taking advantage of a SAYD module one can define cyclic or cocyclic object. To any Hopf–Galois coextensions a SAYD module is associated such that the Hopf cyclic cohomology of the Hopf algebra with coefficients in the SAYD module coincides with the cohomology of the coextension. In other words in the level of cohomology symmetry and the quotient are the only important objects in an extension.

ANDRZEJ SITARZ, Jagiellonian University, Institute of Physics, Reymonta 4, 30-059 Krakow, Poland
The spectral geometry of the Hopf fibration

The examples of both commutative and noncommutative Hopf fibrations ($S^3 \to S^2$ and their noncommutative counterparts) are well studied in the algebraic approach. On the other hand, both three- and two-dimensional noncommutative spheres are nicely described as noncommutative manifolds using Connes formalism of spectral triples. In the talk I shall present the link between the two pictures and discuss the relation between the Dirac operators involved.

PIOTR SOLTAN, Institute of Mathematics of the Polish Academy of Sciences and Department of Mathematical Methods in Physics, Faculty of Physics, University of Warsaw
Homogeneous Spaces for Non-regular Quantum Groups

I will describe two examples of quantum homogeneous spaces of non-regular (non-compact) quantum groups. I will also discuss the action of the quantum groups on their homogeneous spaces and focus on the notion of continuity of actions. The technical tools I will use will include unbounded elements affiliated with C^*-algebras and algebras generated by such elements.

SERKAN SUTLU, University of New Brunswick, Fredericton, Canada
Connes–Moscovici Hopf Algebras Associated To Lie Algebras

In this talk, we first remark a nice duality between the double crossed product and the bi-crossed product Hopf algebras.

In this way we associate a bi-crossed product Hopf algebra to any Lie algebra bi-crossed sum. We show that any such Hopf algebra is equipped with a natural modular pair in involution.

We investigate the relationship between the modules over the Lie algebra in question and those on the bi-crossed product Hopf algebra. As a result we associate a stable anti-Yetter–Drinfeld module over the Hopf algebra to any such module. Finally we show that these Hopf algebras cover all known Hopf algebras constructed from Cartan–Lie pseudogroups.

This is joint work with Bahram Rangipour.

ZHIZHANG XIE, Ohio State University, 231 West 18th Avenue, Columbus, OH 43210, USA

Relative Index Paring and Odd Index Theorem for Even Dimensional Manifolds

The Atiyah–Patodi–Singer twisted index theorem for trivialized flat bundles over odd dimensional manifolds holds for an arbitrary pair of isomorphic bundles. The goal of this talk is to prove an analogue of the theorem for even dimensional manifolds and to obtain explicit formulas for the odd relative index pairing. More specifically, if Y is a closed even dimensional spin manifold and $(U_s)_{0\leq s\leq 1}$ is a smooth path of unitaries over Y, then there is a natural path of lifted projection $(e_s)_{0\leq s\leq 1}$ over $\mathbb{S}^1 \times Y$. Our analogue of Atiyah–Patodi–Singer twisted index theorem takes the following form:

$$\int_0^1 \frac{1}{2}\frac{d}{ds}\eta(e_sDe_s)\,ds = \int_Y \hat{A}(Y)\wedge \mathrm{tch}_\bullet(U_s).$$

BARTOSZ ZIELINSKI, Department of Theoretical Physics II, Uniwersytet Lodzki, Pomorska 149/153, 90-236 Lodz, Poland; Instytut Matematyczny PAN, ul. Śniadeckich 8, 00-956 Warszawa, Poland

Finite closed coverings of compact quantum spaces

We show that a projective space $P^\infty(Z/2)$ endowed with the Alexandrov topology is a classifying space for finite closed coverings of compact quantum spaces in the sense that any such a covering is functorially equivalent to a sheaf over this projective space. In technical terms, we prove that the category of finitely supported flabby sheaves of algebras is equivalent to the category of algebras with a finite set of ideals that intersect to zero and generate a distributive lattice. In particular, the Gelfand transform allows us to view finite closed coverings of compact Hausdorff spaces as flabby sheaves of commutative C^*-algebras over $P^\infty(Z/2)$. As a noncommutative example, we construct from Toeplitz cubes a quantum projective space whose defining covering lattice is free.