Dynamics of a honeybee model on regulation of work distribution

In this talk, we study age-related activities of honeybees (Apis mellifera L.) first by illustrating the life cycle of honeybee which exhibits a combination of individual traits and social cooperation, and then by constructing an age structured model given by a system of difference equations as follows

\[ H_{n+1} = F(H_n) = (A(H_n) - D)H_n + B \quad n \geq 1 \]

where the initial conditions \( H_1 \) is positive, \( B \) represents the broods, \( H_n \) denotes the population of bees divided into \( (k + 1) \) classes with respect to their ages, and \( A(H_n) \) is a lower triangular matrix representing progression or maturation to the next stage of their life involving hive bees and foraging bees, \( D \) indicates the matrix of death rate with subdiagonal entries \( \mu_i \) and others zero. In fact the distribution of most activities and behavioural/physiological maturation are regulated through primer and releaser pheromones from the queen, worker bees, and brood; we include such dynamical organization, as important characteristics for the honey bee, by introducing numerous parameters in the model. Using extensive numerical simulations, we conjecture that this system has a unique positive steady state which is globally asymptotically stable. We are able to prove analytically the global stability (basin of attraction) of feasible solution of such dynamical system with biological relevant special cases.