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On covering triangles by edges

Let $\nu(G)$ denote the maximum number of edge-disjoint triangles in a graph G and $\tau(G)$ denote the minimum number of edges covering all triangles of G. An old conjecture of Tuza states that $\tau(G) \leq 2\nu(G)$ for every graph G. Tuza proved that his conjecture holds for planar graphs, and the result is sharp, since for the graph K_4 we have $\nu(K_4) = 1$ and $\tau(K_4) = 2$. We prove that for every planar graph G with no K_4, $\tau(G) \leq 1.5\nu(G)$, and this result is also sharp.

Let $\tau^*(G)$ denote the minimum total weight of a fractional covering of its triangles by edges. Krivelevich proved the fractional version of Tuza's conjecture, that $\tau^*(G) \leq 2\nu(G)$ for every graph G. We give a stability version of this result by showing that if a graph G has $\tau^*(G) \geq 2\nu(G) - x$, then G contains $\nu(G) - \lfloor 10x \rfloor$ edge-disjoint K_4-subgraphs plus $\lfloor 10x \rfloor$ additional edge-disjoint triangles.

This represents joint work with A. Kostochka and S. Thomassé.