Algebraic Geometry, Non-commutative Algebra and Derived Categories
Géométrie algébrique, algèbre non commutative et catégories dérivées
(Org: **Colin Ingalls** (UNB))

CHRIS BRAV, Toronto

PINAR COLAK, Simon Fraser University, 8888 University Drive, Burnaby, BC V5A 1S6
Two-sided Ideals in Leavitt Path Algebras

Leavitt path algebras are a natural generalization of the Leavitt algebras, which are a class of algebras introduced by Leavitt in 1962. For a directed graph E, the Leavitt path algebra $L_K(E)$ of E with coefficients in K has received much recent attention both from algebraists and analysts over the last decade. So far, some of the algebraic properties of Leavitt path algebras have been investigated, including primitivity, simplicity and being Noetherian. We explicitly describe two-sided ideals in Leavitt path algebras associated with a row-finite graph. Our main result is that any two-sided ideal I of a Leavitt path algebra associated with a row-finite graph is generated by elements of the form $v + \sum_{i=1}^{n} \lambda_i g^i$, where g is a cycle based at vertex v. We use this result to show that a Leavitt path algebra is two-sided Noetherian if and only if the ascending chain condition holds for hereditary and saturated closures of the subsets of the vertices of the row-finite graph E. Moreover, we show that this result can be used to unify and simplify many known results for row-finite Leavitt path algebras.

MICHELE D'ADDERIO, University of California San Diego
Entropy in algebras

We introduce the notion of entropic Følner function for algebras and we show its relation with the isoperimetric profile and the lower transcendence degree.

We explain how entropic technique like Shannon inequality can lead to a theorem on the lower transcendence degree of domains and division algebras.

We finally discuss its relation with some old conjectures of M. Artin, L. Small and J. Zhang.

KEN GOODEARL, University of California at Santa Barbara
Zariski topology in quantized coordinate rings

The topological structures of the prime and primitive spectra of a generic quantized coordinate ring A are known "piecewise". Assuming the base field is algebraically closed, there is an algebraic torus H acting on A in such a way that there are only finitely many H-orbits in $\operatorname{prim} A$, each of which is homeomorphic to an affine variety (actually, another torus); similarly, $\operatorname{spec} A$ is a finite disjoint union of locally closed subsets homeomorphic to the prime spectra of algebraic varieties. What is missing is any description of the topological relationships among the above-mentioned pieces of $\operatorname{spec} A$ and $\operatorname{prim} A$. We will discuss a framework for such a description, present it in detail for quantum GL_2, and raise the question of how it might relate to classical algebraic geometric structures.

ED GREEN, Virginia Tech
When are selfinjective algebras Koszul?

Let A be a (finite dimensional) graded connected selfinjective algebra. In this talk, I will discuss sufficient conditions on A that imply that A is Koszul. We present two different results. The first result restricts the Poincaré series and requires the existence of a copoint module. The second result is homological in nature and requires the existence of a system of modules (copoint, coline, etc) that are interrelated by short exact sequences.

ELLEN KIRKMAN, Wake Forest University
Invariants of AS-Regular Algebras: Complete Intersections

Let G be a finite group acting on an Artin–Schelter regular $\mathbb{C}$-algebra A. Extending results of Watanabe we give conditions when the invariant subring A^G is an Artin–Schelter Gorenstein algebra. When $A = \mathbb{C}[x_1, \ldots, x_n]$ Gordeev (1986) and Nakajima (1984) independently determined when A^G is a complete intersection. We discuss extending these results to other Artin–Schelter regular algebras.

BASIL NANAYAKKARA, University of New Brunswick, Fredericton, NB, Canada
Brauer pairs and terminal resolutions

Given a quasi-projective variety X and an element α in the 2-torsion part of the Brauer group of the function field of X, the pair (X, α) is called a *Brauer pair*. There is a notion of *discrepancy* of a Brauer pair which is similar to the notion of discrepancy of a logarithmic pair that one encounters in Mori's minimal model program. (We will discuss this notion in the talk.) A Brauer pair is called a *terminal pair* if its discrepancy is positive.

If there is a birational morphism $f\colon Y \longrightarrow X$ then we can view an element α in the Brauer group $\mathrm{Br}\big(k(X)\big)$ of the function field $k(X)$ as an element in $\mathrm{Br}\big(k(Y)\big)$ via the isomorphism $f^*\colon k(X) \longrightarrow k(Y)$. We will show that given a Brauer pair (X, α) with X 3-dimensional, there is a terminal pair (Y, α) with a birational morphism $f\colon Y \longrightarrow X$. In other words, every 3-dimensional Brauer pair admits a terminal resolution.

ADAM NYMAN, Western Washington University, 516 High St., Bellingham, WA 98225
Non-commutative quadrics are ruled

Non-commutative quadrics, defined and classified by M. Van den Bergh, are non-commutative deformations of $\mathbb{P}^1 \times \mathbb{P}^1$. They are classified in terms of their point schemes C and associated commutative geometric data.

We describe progress towards proving that a non-commutative quadric with C a smooth genus 1 curve is a non-commutative ruled surface over $\mathbb{P}^1$.

DAVID SALTMAN, IDA–Princeton
Embedding Division Algebras

Division rings are noncommutative fields meaning associative rings where every nonzero element has a multiplicative inverse. Division algebras are division rings finite dimensional (or just finite) over their centers and are the basic elements in Brauer groups. Many years ago P. M. Cohn proved that any two division rings of the same characteristic embed in a third division ring. If one applies this to division algebras, the new third division ring is huge and is certainly NOT finite over its center. Thus Lance Small asked whether two division algebras of the same characteristic could be embedded in a third division algebra. We will show the answer is "no", but yes if we require the division algebras be finitely generated over the same prime or other perfect field. The tools we will use involve generalized Severi–Brauer varieties (forms of Grassmanns) and some old index reduction results that, in one form, involve Moody's Theorem.

This is joint work with Louis Rowen.

GORDANA TODOROV, Northeastern University, Boston, MA 02115, USA
Generalized cluster categories and c-sortable words

Categories with some of the essential properties of cluster categories (triangulated, 2 Calabi–Yau) were already defined by several authors; in particular, certain subcategories of modules over preprojective algebras associated to the words in the Coxeter group are such categories.

On the other hand, the basic construction of cluster categories as orbit categories of the derived categories of the module categories of algebras of global dimension 1, was generalized to the global dimension 2 algebras, by considering triangulated hull of the above mentioned orbit category in this case.

We construct a triangle equivalence between the 2-Calabi–Yau triangulated categories associated to the words in the Coxeter group as mentioned above, and generalized cluster categories; we construct appropriate algebras of global dimension 2.

Joint work with C. Amiot, O. Iyama and I. Reiten, arXiv:math. RT/1002.4131.

HOKUTO UEHARA, Tokyo Metropolitan University, Tokyo, Japan
Tilting generators via ample line bundles

It is known that a tilting generator on an algebraic variety X gives a derived equivalence between X and a certain non-commutative algebra. I explain a method to construct a tilting generator from an ample line bundle.

My talk is based on a joint paper with Y. Toda in Advances in Mathematics **233**(2010).

XIANDE YANG, Department of Mathematics and Statistics, University of New Brunswick
On Morphic Trivial Extension of a Commutative Domain

An associative ring R with unit is left morphic if for every element $a \in R$, there exists some $b \in R$ such that the left annihilators $l_R(a) = Rb$ and $l_R(b) = Ra$. Analogously, we can define right morphic and morphic rings. For a commutative domain R, we prove that the trivial extension $R \ltimes M$ is morphic if and only if R is Bezout and $M \cong \frac{Q}{R}$. This positively answered a question of a recent paper.