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Coarsening polyhedral complexes

A polyhedron is an intersection of finitely many halfspaces. A polyhedral complex is a finite collection of polyhedra which intersect “nicely”. The support of a polyhedral complex is the union of all of its polyhedra. A polyhedral complex C' is said to coarsen another complex C if every polyhedron in C' is a union of polyhedra in C .

I will describe a local codimension-2 criterion which characterizes coarsenings of a polyhedral complex with convex support. The criterion broadly generalizes a result of Morton, Pachter, Shiu, Sturmfels and Wienand, which identifies the “semigraphoids” of nonparametric statistics with coarsenings of the polyhedral complex (fan) defined by the braid arrangement. The criterion is also closely related to my earlier work on fans defined by lattice congruences of the weak order. I will sketch the proof, which makes use of a generalization of Tits’ solution to the Word Problem and a surprising shortcut for checking whether a set of polyhedra is a polyhedral complex. Time allowing, I will discuss a byproduct of the proof: Given a union U of polyhedra such that the interior of U is connected, I give a local criterion for deciding when U is convex.