Reaction-Diffusion Systems and Their Applications
Systèmes de réaction-diffusion et leurs applications
(Org: **David Iron** (Dalhousie), **Theodore Kolokolnikov** (Dalhousie) and/et **Chunhua Ou** (Memorial))

DAVE AMUNDSEN, Carleton University
Symmetry breaking and pattern formation in a model for neural development

At a cellular level, the development of the nervous system and in particular axons, neurons and neuroblastoma cells is governed by a complex chemical and biological mechanism. A well-known model for this mechanism involves what are understood to be the key elements in this process, called Retinoic Acid and Notch. The model seeks to capture their feedback and interaction both inside and outside the cell. Communication across the cell membrane is achieved via so-called signaling pathways which are assumed independent. However while the model has proven consistent in a number of respects, recent evidence suggests that in fact the signaling pathways may interact with one another. The present study seeks to model and investigate the impact of this interaction in terms of the underlying mechanisms for symmetry breaking and pattern formation. Generalizations to a wider class of Reaction Diffusion equations will be discussed and numerical examples presented.

This is joint work with Majid Bani.

THOMAS HILLEN, University of Alberta
Merging and Emerging Patterns in Chemotaxis

The study of pattern formation for chemotaxis PDEs (partial differential equations) started with the identification of blow-up solutions. If, however, the model is adapted to allow for global existence of solutions, then another interesting pattern formation process arises. Local maxima form and they show an interaction of merging (two local maxima coagulate) or emerging (a new maximum is formed). This dynamics can lead to steady states, periodic solutions or to (what we think is) chaotic behavior. I will show that this pattern interaction is very typical for a wide variety of chemotaxis models and I will discuss possible ideas on how to understand this complicated pattern interaction.

Joint work with K. Painter and Z. Wang.

HUAXIONG HUANG, York University, Toronto, ON
Models for Bread Baking: Moisture Transport and Diffusive Instability

In this talk we discuss two related multiphase models for simultaneous heat and mass transfer process during bread baking. Our main objective is to provide an explanation and a remedy to the observed erroneous and/or divergent results associated with the instantaneous phase change model used in the literature. We propose a reaction-diffusion model based on the Hertz–Knudsen equation, where the phase change is not instantaneous but determined by an evaporation/condensation rate. A splitting scheme is designed so that a relation between these two models can be established and the non-intuitive numerical instability associated to the instantaneous phase change model can be identified and eliminated through the reaction-diffusion model. The evaporation/condensation rate is estimated from balancing these two models and reasonable and consistent results are produced by using the estimated rate. For the evaporation/condensation rate beyond the estimated value oscillation, solutions with multiple regions of dry and two-phase zones is observed. We show that these are caused by an instability intrinsic to the model (which we call diffusive instability) and the effect of the diffusive instability to the bread-baking simulation is also explained through a linear stability analysis and supported by numerical tests.

This is a joint work with P. Lin and W. Zhu.

DAVID IRON, Dalhousie University
Dynamics of a Two-Spike Solution to the Gierer–Meinhard System in Two-Dimensions

In this talk I will discuss a two-spike solution to the Gierer–Mienhardt system posed in two-dimensions. I will construct a differential equation governing the separation of the spikes. I will then show that if the distance between the spikes is below a critical value, one spike will be destroyed. If the distance between the spikes is above this critical distance a Hopf bifurcation may occur as the spikes move apart.

MEI MING, Champlain College & McGill University
Stability of travelling waves for time-delayed reaction-diffusion equations

This is the series of study on the stability of traveling wavefronts of reaction-diffusion equations with time delays. In this talk we will consider a local and nonlocal time-delayed reaction-diffusion equation, respectively. When the initial perturbation around the traveling wave decays exponentially as $x \to -\infty$ (but the initial perturbation can be arbitrarily large in other locations), we prove the asymptotic stability of all traveling waves for the reaction-diffusion equation, including even the slower waves whose speed are close to the critical speed. This essentially improves the previous stability results for the faster speed waves with small initial perturbation. The approach we use here is the technical weighted energy method, but the weight function is more tricky to construct due to the property of the critical wavefront, and the difficulty arising from the nonlocal nonlinearity is also overcome. Finally, by using the Crank–Nicolson scheme, we present some numerical results which confirm our theoretical study.

DMITRY PELINOVSKY, Department of Mathematics, McMaster University
Advection-diffusion equations with sign-varying diffusion

We study the spectrum of the advection-diffusion operator

$$L = -\partial_\theta - \epsilon\partial_\theta(\sin\theta\partial_\theta)$$

subject to the periodic boundary conditions on $[-\pi, \pi]$. We prove that the operator is closed in $L^2_{\rm per}(-\pi,\pi)$ with the domain in $H^1_{\rm per}(-\pi,\pi)$ for $|\epsilon| < 2$, its spectrum consists of an infinite sequence of isolated eigenvalues and the set of corresponding eigenfunctions is complete. By using numerical approximations of eigenvalues and eigenfunctions, we show that all eigenvalues are simple, located on the imaginary axis and the angle between two subsequent eigenfunctions tends to zero for larger eigenvalues. As a result, the complete set of linearly independent eigenfunctions does not form a basis in $L^2_{\rm per}(-\pi,\pi)$.

MICHAEL WARD, University of British Columbia, Vancouver, BC
Diffusion on a Sphere with Localized Traps: Mean First Passage Time, Eigenvalue Asymptotics, and Fekete Points

A common scenario in cellular signal transduction is that a diffusing surface-bound molecule must arrive at a localized signaling region on the cell membrane before the signaling cascade can be completed. In order to determine the time-scale for this process, we calculate asymptotic results for the mean first passage time for a diffusing particle confined to the surface of a sphere in the presence of multiple partially absorbing traps of small radii. In addition, asymptotic results are given for the related problem of calculating the mean first passage time for a diffusing particle inside a sphere with small traps on an otherwise reflecting boundary condition. The asymptotic analysis relies on detailed properties of certain Green's functions related to the sphere. The asymptotic results are shown to compare favorably with full numerical results.

Joint work with Dan Coombs (UBC), Ronny Straube (Max Planck Institute, Magdeburg).

ZOU XINGFU, University of Western Ontario
Co-invasion waves of a reaction diffusion model for competing pioneer and climax species

We consider a reaction diffusion model for competing pioneer and climax species. A previous work has established the existence of traveling wave fronts connecting two competition-exclusion equilibria in certain range of the parameters, while in this paper, we explore the possibility of traveling wave fronts connecting the pioneer-invasion-only equilibrium and the co-invasion equilibrium. By combining the Schauder's fixed point theorem with a pair of the so-called desired functions, we show that the model does support such co-invasion waves in some other ranges of parameters. We also determine the minimal speed for such co-invasion waves in terms of the parameters, and discuss some biological implications and significance of the results.

This a joint work with Zhaohui Yuan.

ZHIYONG ZHANG, University of Alberta, 632 CAB, Math Department, Edmonton, Alberta, T6G 2G1
A analysis of a biosensor model

We analyze a nonlinear biosensor model involving a parabolic equation with Robin boundary condition and an ODE. The existence and uniqueness of the solution is obtained by topological methods. The long-time behavior and system case are also discussed. A finite volume method is applied and convergence, stability and error estimates, and some numerical simulations are obtained for the approximate solution.

The work is done with Dr. Allegretto Walter and Dr. Lin Yanping.

XIAO-QIANG ZHAO, Memorial University of Newfoundland, St. John's, NL, A1C 5S7
Monotone Wavefronts for Partially Degenerate Reaction-Diffusion Systems

In this talk, I will report our recent research on monotone wavefronts for cooperative and partially degenerate reaction-diffusion systems. The existence of monostable wavefronts is established via the vector-valued upper and lower solutions method. It turns out that the minimal wave speed of monostable wavefronts coincides with the spreading speed. The existence of bistable wavefronts is obtained by the vanishing viscosity approach combined with the properties of spreading speeds in monostable cases.

This is joint work with Jian Fang.