
Mathematical Physics
Physique mathématique
(Org: **Marco Merkli** and/et **Chris Radford** (Memorial))

IVAN BOOTH, Memorial University, St. John's, Newfoundland and Labrador

Supersymmetric isolated horizons

Black holes that are in equilibrium with their surroundings are necessarily bounded by isolated horizons: non-expanding null hypersurfaces whose cross-sections are compact spacelike surfaces. Over the last decade this formalism has become a standard tool used in the study of classical and quantum gravity. In this talk I will discuss a recent application of isolated horizons to the study of supersymmetric black holes. Results include the fact that supersymmetric isolated horizons are necessarily extremal and have severely restricted geometry, electromagnetic charges and angular momentum.

RICHARD FROESE, University of British Columbia

Absolutely continuous spectrum for a random potential on a tree with strong transverse correlations and large weighted loops

We consider random Schrödinger operators on tree graphs and prove absolutely continuous spectrum at small disorder for two models.

The first model is the usual binary tree with certain strongly correlated random potentials. These potentials are of interest since for complete correlation they exhibit localization at all disorders. In the second model we change the tree graph by adding all possible edges to the graph inside each sphere, with weights proportional to the number of points in the sphere.

This is joint work with David Hasler and Wolfgang Spitzer.

GILAD GOUR, University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N 1N4

On the additivity conjecture in quantum information

Quantum information science, an interface area of mathematics, physics and computing science, is concerned with the manipulation, computation and communication of information, where the information is encoded in two (or more) level quantum systems called “qubits”, unlike classical information, which is encoded in Boolean “bits”. The devices used in this science are governed by the principles of quantum mechanics, which opens the possibility for a large range of applications. In this talk I will discuss the long standing additivity conjecture that the minimum entropy output of a completely positive trace preserving linear map, as measured using the von Neumann entropy, is additive under taking tensor products. After enormous efforts by the most experts in the field during the last 12 years, this conjecture has been recently proven to be false. Here I will present a slight modification to this (false) conjecture and discuss some recent progress and future directions.

This talk is based on a joint work with Shmuel Friedland, Aidan Roy, and Jon Yard.

CODY HOLDER, University of Alberta

Liouvillian Quasinormal Modes of Black Holes

The ordinary differential equations governing the radial parts of the perturbations of Kerr–Newman black holes admit Liouvillian (closed-form) solutions for a discrete set of explicit frequencies. For configurations of the black hole charge and angular momentum in which the angular parts of the perturbations approach the spherical harmonics we find Liouvillian solutions satisfying the boundary conditions for quasinormal modes.

RICHARD LAVINE, University of Rochester, Rochester, NY 14627, USA

Time of Decay in Quantum Mechanics

Quantum mechanics prescribes the probabilities of possible outcomes when a measurement is made at a particular time. The times when events like decay occur are also random, but the theory provides no direct prescription for their probabilities, even though these times are often observed in experiments. We study a model where a system with a decaying state is coupled with a detector. If by itself the initial state decays exponentially, the detected time of decay also has an exponential distribution. If the initial state is in the domain of its Hamiltonian, precisely exponential decay is impossible, and a detector which is too precise retards the decay, i.e., the quantum Zeno effect occurs. But if the decay is approximately exponential, and the detector is not overly precise, the detected time also is approximately exponential.

KARL-PETER MARZLIN, St Francis Xavier University
Moyal phase-space analysis of nonlinear optical Kerr media

The Moyal equation of a quantum observable corresponds to a phase space representation of its Heisenberg equation of motion. Because the latter has a close relation to the corresponding classical dynamics, the Moyal method is ideally suited to study the transition from classical to quantum behaviour in a system. The Moyal representation is related to the Wigner function like the Heisenberg picture is related to the Schroedinger picture. Unfortunately the Moyal equation is difficult to solve so that only few exact solutions are known.

We have studied nonlinear optical self-phase modulation of Kerr type using the Moyal equation for a single optical field mode. An exact solution for the annihilation operator is found. The phase space representation of this operator is related to the classical field amplitude by a complex factor that shows characteristic singularities in time. We show that these singularities disappear in the classical limit and demonstrate how the uncertainty relation prevents that observable quantities are affected by it.

MARCO MERKLI, MUN
Evolution of entanglement and coherence

We consider a system $S = S_1 + S_2$ of two spins $1/2$ (qubits) interacting with several thermal reservoirs. Each spin $j = 1, 2$ is coupled to an individual reservoir R_j and the two spins interact collectively with a third reservoir R . All reservoirs are at the same temperature and do not interact directly. Each interaction between a spin and a reservoir has two channels, an energy-conserving and an energy-exchange one.

We analyze decoherence, thermalization and disentanglement of the system S . We show that due to the energy-exchange interactions, the system has a *finite disentanglement time*, we estimate that time and compare it to decoherence times.

This is joint work with Kassu Gebresellasie.

DMITRY PELINOVSKY, Department of Mathematics, McMaster University
Asymptotic stability of small bound states in the discrete NLS equation

Asymptotic stability of small bound states in one dimension is proved in the framework of a discrete nonlinear Schrödinger equation with septic and higher power-law nonlinearities and an external potential supporting a simple isolated eigenvalue. The analysis relies on the dispersive decay estimates from Pelinovsky and Stefanov (2008) and the arguments of Mizumachi (2008) for a continuous nonlinear Schrödinger equation in one dimension. Numerical simulations suggest that the actual decay rate of perturbations near the asymptotically stable solitons is higher than the one used in the analysis.

This is a joint work with A. Stefanov and P. Kevrekidis.

TUOC V. PHAN, University of British Columbia
Stable Directions for Degenerate Excited States of Nonlinear Schrödinger Equations

We consider the nonlinear Schrödinger equations $i\partial_t\psi = H_0\psi + \lambda|\psi|^2\psi$ in \mathbb{R}^3 where $H_0 = -\Delta + V$ and λ is a given constant which can be positive or negative. Assume that the potential V is radial and decays sufficiently fast at infinity. Assume also

that the linear Hamiltonian H_0 has only two discrete eigenvalues $e_0 < e_1 < 0$ where e_0 is simple and e_1 has multiplicity 3. We show that there exist three branches of excited states and for certain finite codimension subset in the space of initial data, we construct solutions ψ converging to each of these excited states in both non-resonant and resonant cases.

This is a joint work with Stephen Gustafson.

YVAN SAINT-AUBIN, Université de Montréal, CP 6128 Centre-ville, Montréal

Critical exponents for the homotopy of Fortuin–Kasteleyn clusters on a torus

We study the critical behavior of statistical lattice models in 2d using the homotopy of their Fortuin–Kasteleyn (FK) cluster. A FK cluster on a torus is said to be of type $\{a, b\}$, $a, b \in \mathbb{Z}$, if it is possible to draw a curve belonging to the cluster that winds a times around the first cycle of the torus as it winds $-b$ times around the second. Even though the Q -Potts models make sense only for Q integers, they can be included into a family of models parametrized by $\beta = \sqrt{Q}$ for which the FK clusters can be defined for any real $\beta \in (0, 2]$. For this family, we study the probability $\pi(\{a, b\})$ of a given type of clusters as a function of the torus modular parameter $\tau = \tau_r + i\tau_i$. We compute the asymptotic behavior of some of these probabilities as the torus becomes infinitely thin. Exponents describing these behaviors are defined and related to weights $h_{r,s}$ of the extended Kac table for r, s integers, but also half-integers. Numerical simulations are also presented.

Joint work with Alexi Morin-Duchesne.

ROMAN SMIRNOV, Dalhousie University, Department of Mathematics and Statistics, Halifax, NS, B3H 3J5

Hamilton–Jacobi theory in Minkowski space via Cartan geometry

A complete solution to the problem of orthogonal separation of variables of the Hamilton–Jacobi equation in three-dimensional Minkowski space is obtained. The solution is based on the underlying ideas of Cartan geometry and ultimately developed into a general new algorithm that can be employed in the study of Hamiltonian systems defined by natural Hamiltonians within the framework of Hamilton–Jacobi theory. To demonstrate its effectiveness, we investigate, from this viewpoint, the Morosi–Tondo integrable system derived as a stationary reduction of the seventh-order Korteweg–de Vries flow to show explicitly that the system in question is an orthogonally separable Hamiltonian system. The latter result is a new characterization of the Morosi–Tondo system.

This is joint work with Joshua Horwood and Raymond McLenaghan.

REINEL SOSPEDRA, University of Victoria, Mathematics and Statistics, SSM Building A425, 3800 Finnerty Rd., Victoria, BC, V8P 5C2

Global classical solutions to the 3D relativistic Vlasov–Maxwell system with bounded spatial density

The relativistic Vlasov–Maxwell system (RVM in short) is a kinetic model that arises in plasma physics and describes the time evolution of a collisionless plasma whose particles interact through the self-induced electromagnetic field. The plasma is assumed to be at high temperatures, thus the particles may travel at speeds comparable to the speed of light. The main open problem concerning this system is to prove whether or not classical solutions develop singularities in finite time. Glassey and Strauss established the existence and uniqueness of local in time classical solutions for smooth and compactly supported initial data. They showed that such solutions can be continued globally in time provided the momenta of the particles are controlled. Subsequently, they proved that such control is achieved if their kinetic energy density remains bounded for bounded times. In this talk, we show that the latter assumption can be weakened to the boundedness of the spatial density.

SHANNON STARR, University of Rochester, Rochester, NY 14627, USA

CLT in the Annealed Region of the Diluted Mean-Field Spin Glass

This talk represents joint work with Brigitta Vermesi. Using Stein's method, Chatterjee and Crawford proved a CLT for the energy density in the high temperature region of the Sherrington–Kirkpatrick mean-field spin glass model. Using Chen's

extension of Stein's method, we proved CLT's for the dilute mean-field spin glass model known as the Viana–Bray model, where only a fraction of edges are present, leaving a random graph. The fluctuations are larger and some new features appear.

HOLGER TEISMANN, Acadia University
Controllability of nonlinear Schrödinger equations

In this talk we will review some recent results on the (local exact) controllability of (certain) nonlinear Schrödinger equations with interior, boundary, and bilinear controls. Some applications to quantum control systems will be discussed.

FRED TING, Lakehead University, 955 Oliver Road, Thunder Bay, Ontario, P7B 5E1
Multi-Vortex Pinning to Ginzburg–Landau equations with External Potential in the Plane

We study the existence of multi-vortex solutions to the Ginzburg–Landau equations external potential in \mathbb{R}^2 . We show that if the external potential is *strong enough* (in some sense) and has *widely spaced* critical points, then there exists multi-vortex solutions centered near the critical points of the external potential.

This is joint work with Aaron Pakylak.

ROBERT VAN DEN HOOGEN, St. Francis Xavier University
Parallel Transport, Non-local Calculus, and the Smoothing of Spacetime

A procedure is developed to average tensor fields over simply connected differentiable manifolds. Parallel transport in metric affine spaces is reviewed and arguments are made to ensure that an averaging procedure be well defined. We argue that in a Riemannian space (V_n) only parallel transport along geodesics provides for a well-defined averaging procedure. Alternatively, if one wishes to develop a well-defined averaging procedure that is independent of the path used for parallel transport, then one must use a Weitzenböck connection on a flat manifold (a space of distant parallelism or W_n). With these preliminaries, we define the average of a tensor field over a finite region in both V_n and W_n spaces, and conclude with comparisons to some of the other averaging procedures.

BUKS VAN RENSBURG, Math. & Stats., York University, Toronto, ON, M3J 1P3
Monte Carlo Sampling of Self-Avoiding Walks and Polygons

A Monte Carlo method for sampling self-avoiding walks and polygons is presented. The method (GAS, for Generalised Atmospheric Sampling), samples polygons or walks along weighted sequences by implementing elementary moves generated by the positive, negative and neutral atmospheric statistics of the polygons or walks. GAS is a self-tuning algorithm which samples from uniform distributions over lengths of polygons and walks in an interval $[0, n_{\max}]$ - this implementation is called "flatGAS" (flat histogram GAS).

In the case of polygons, states along a sequence realised by GAS are weighted such that the average weight of states of length n edges is proportional to p_n (the number of polygons of length n). Hence, GAS is an approximate enumeration method, and I shall present data estimating the numbers of cubic lattice polygons of specified knot type.

VITALI VOUGALTER, University of Toronto, Department of Mathematics, Toronto, ON, M5S 2E4
Solvability conditions for some non-Fredholm operators

We obtain solvability conditions for some elliptic equations involving non-Fredholm operators with the methods of spectral theory and scattering theory for Schroedinger-type operators.

ERIC WOOLGAR, University of Alberta
The Ricci flow and Bartnik's quasi-local mass

Consider a Riemannian manifold M with an asymptotic end and a compact (i.e., inner) boundary. On this boundary, fix the boundary metric and mean curvature. Bartnik's "static minimization conjecture" is that, of all asymptotically flat Riemannian metrics on M with

- (i) nonnegative scalar curvature,
- (ii) containing no minimal hyperspheres, and
- (iii) inducing the given metric and mean curvature on the boundary,

there will be a metric which minimizes the ADM mass of M , and that this metric obeys the static Einstein equations on M . B. List in his PhD thesis described a geometric flow, now recognized to be a certain Ricci flow, which leads to an approach to this problem. I will describe a study of List's flow in the rotationally symmetric case.

This is joint work with T. Oliynyk and L. Gulceva.