
MATTHIAS KUENZER, University of Aachen, Germany

A Fourier–Hopf inversion and spectral sequences

Let H be a Hopf algebra over a commutative ring R . Let K be a normal Hopf subalgebra of H . Write $\bar{H} := H/HK^+$. Suppose given an H -module M . Then $\mathrm{Hom}_K(H, M)$ and $\mathrm{Hom}_R(\bar{H}, M)$ are isomorphic as left \bar{H} -modules, the isomorphism resembling a bit a Fourier inversion. Under some projectivity assumptions, this can be used to show that $\mathrm{Hom}_K(H, M)$ is $\mathrm{Hom}_{\bar{H}}(R, -)$ -acyclic. Therefore, in this situation there are two spectral sequences,

(1) the Grothendieck spectral sequence, working with injective Cartan–Eilenberg resolutions, and

(2) a more pedestrian one, working with a tensor product of projective resolutions.

Moreover, (1) and (2) are isomorphic, so in particular, they converge both to $\mathrm{Ext}_H^*(R, M)$. This specialises to groups (Lyndon–Hochschild–Serre) and to Lie algebras (Hochschild–Serre).