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Hopf algebras and root systems

The first fundamental problem in the classification theory of pointed Hopf algebras is to understand Nichols algebras (or quantum symmetric algebras). Prominent examples of Nichols algebras are the $+$-parts of the quantum groups of semisimple Lie algebras. More concretely a very general and basic question is the following: What is the structure of the Nichols algebra of a finite direct sum V of finite-dimensional irreducible Yetter–Drinfeld modules (over any Hopf algebra with bijective antipode)? It turns out that there is a rich combinatorial context which can be used to answer this question.

In joint work with I. Heckenberger we associate a generalized root system to a Nichols algebra of this type (like the root system of a Kac–Moody Lie algebra), and in recent joint work with N. Andruskiewitsch and I. Heckenberger we define reflections of a Weyl groupoid of the Nichols algebra of V (like the Weyl group of a generalized Cartan matrix). I will give an introduction to these new developments and to some of their applications.