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On Orders of Torsion Units in Integral Group Rings of Sporadic Simple Groups

Let $V(\mathbb{Z}G)$ be the normalized unit group of the integral group ring $\mathbb{Z}G$ of a finite group G. The long-standing conjecture of H. Zassenhaus (ZC) says that every torsion unit from $V(\mathbb{Z}G)$ is conjugate within the rational group algebra $\mathbb{Q}G$ to an element of G.

W. Kimmerle proposed to relate (ZC) with some properties of graphs associated with groups. The Gruenberg–Kegel graph (or the prime graph) of the group G is the graph with vertices labelled by the prime divisors of the order of G with an edge from p to q if and only if there is an element of order pq in the group G. Then Kimmerle's conjecture (KC) asks whether G and $V(\mathbb{Z}G)$ have the same prime graph.

We started the project of verifying (KC) for sporadic simple groups, using the Luthar–Passi method with recent extensions by M. Hertweck as a main tool. Now we are already able to report that (KC) holds for the following thirteen sporadic simple groups:

- Mathieu groups $M_{11}, M_{12}, M_{22}, M_{23}, M_{24}$;
- Janko groups J_1, J_2, J_3;
- Held, Higman–Sims, McLaughlin, Rudvalis and Suzuki groups.

In my talk I will summarise known information about orders and partial augmentation of these groups, explain enhancements of the Luthar–Passi method that were developed during the project, and highlight some challenges arising from the remaining sporadic simple groups.

Joint work with Victor Bovdi, Eric Jespers, Steve Linton, Salvatore Siciliano et al.