A. C. SOUZA-FILHO, Universidade de São Paulo

Free Groups in Quaternion Algebras

Hyperbolic groups were first studied by M. Gromov. The Flat Plane Theorem states that if a group is hyperbolic it has no subgroup isomorphic to $\mathbb{Z} \times \mathbb{Z}$. This result has an important role in the classification of the groups G and the rational quadratic extensions $K = \mathbb{Q}\sqrt{-d}$ such that the group of normalized units $V = \mathcal{U}_1(\mathfrak{o}_K[G])$ is a hyperbolic group, where $\mathfrak{o}_K$ is the integral algebraic ring of K. If G is a non-abelian group, Juriaans, Passi and Souza Filho proved that V is hyperbolic if, and only if, G is the quaternion group of order 8 and d is a positive square free integer such that $d \equiv 7 \pmod{8}$. In this case, the quaternion algebra $\mathcal{H}(K)$ is a division ring. Since the unit group of the $\mathbb{Z}$-order $\Gamma = \mathcal{H}(\mathfrak{o}_K)$ is hyperbolic, we also show that for a suitable pair of units u, v of Γ there is an integer m such that the group generated by the powers u^m, v^m is a free subgroup of Γ of rank two. Juriaans and Souza Filho extended this result and show that, in fact, the theorem is true for any positive square free integer d. We also prove that the power m is equal to 1, except in the case $d = 2$ which $m = 2$.

In this talk we shall communicate results obtained by S. O. Juriaans and A. C. Souza Filho in which the units u, v are constructed by the methods due to S. O. Juriaans, I. B. S. Passi and A. C. Souza Filho.