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On torsion units in $\mathbb{Z}\,\mathrm{Sz}(q)$

Let $V(\mathbb{Z}G)$ denote the group of normalized units of the integral group ring $\mathbb{Z}G$, i.e., the units with coefficient sum 1. A more than 30-year-old conjecture of Hans Zassenhaus states:

(ZC1) For a finite group G every torsion unit u in $V(\mathbb{Z}G)$ is conjugate within $\mathbb{Q}G$ to an element of G.

This conjecture has been verified for some classes of groups, but only for very few non-solvable groups. Luthar, Passi and Hertweck developed a method to deal with this conjecture with arithmetical means using the characters of the groups. With the aid of this tool the following for the smallest group of the series of the simple Suzuki groups $\mathrm{Sz}(q)$ is shown: The orders of torsion units in $V\big(\mathbb{Z}\,\mathrm{Sz}(8)\big)$ coincide with orders of elements of $\mathrm{Sz}(8)$; if the order is 2, 5 or 13 the elements are conjugate by units of $\mathbb{Q}\,\mathrm{Sz}(q)$ to group elements. This gives an affirmative answer to a question of Kimmerle, namely if the prime graphs of $V(\mathbb{Z}G)$ and G coincide, in this case.

Recently Hertweck, Höfert and Kimmerle showed that for all prime powers q and all primes r the finite r-subgroups of $V\big(\mathbb{Z}\,\mathrm{PSL}(2\,;\,q)\big)$ are isomorphic to subgroups of $\mathrm{PSL}(2\,;\,q)$. Using similar methods and the above result we obtained that for a finite minimal simple group G and a prime r any elementary abelian r-group $H \leq V(\mathbb{Z}G)$ is isomorphic to a subgroup of G, except possibly the case $G \cong \mathrm{PSL}(3\,;\,3)$ and $H \cong C_3 \times C_3 \times C_3$.