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Detours in the Development of Mathematical Thinking

The study of children's mathematical thinking provides a valuable window on cognitive development during childhood, as well as insights into how to optimize the ways in which children learn mathematics. The common view is that learning mathematics is an incremental process in which older children develop new and more powerful concepts that incorporate or replace previous concepts. We have studied the development of two mathematical concepts in some detail: inversion, the principle that $a+b-b$ must equal a; and equivalence, the concept that two sides of an equation must represent the same quantity. Contrary to the common view, many older children have considerable difficulty understanding or using these concepts in arithmetic despite the fact that preschool and early elementary school children appear to use these very concepts under certain conditions. These detours in development appear to be due to difficulties that children experience—and that we create—when we impose the symbol-based mathematical tools of our culture onto the informally acquired, intuitive mathematical knowledge of young children.