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Additive group actions associated to derivations of $R[X,Y,Z]$ with a slice

This talk features a simple family of locally nilpotent R-derivations of $R[X,Y,Z]$ with a slice, where $R=\mathbb{C}[a,b]$. Equivalently, this is a family of $\mathbb{G}_a$-actions on $\mathbb{A}^5$ such that $\mathbb{A}^5 = V \times \mathbb{A}$, where V is the variety defined by the algebra of invariants, and $\mathbb{G}_a$ acts by translation. We show that V is an $\mathbb{A}^2$-fibration over $\mathbb{A}^2$, but it is unknown whether this is a trivial fibration. Note that V has the form $\mathrm{Spec}(B/sB)$, where $B=R[X,Y,Z]$ and $s \in B$ is the corresponding slice. We give a method for finding $f \in B$ of degree smaller than s such that B/fB and B/sB are isomorphic as fibrations. However, it is not known whether f is a slice for any locally nilpotent derivation of B. These examples are motivated by the Vénéreau polynomials $v \in L = \mathbb{C}[x,y,z,u]$. It was shown by the author that, if $K=\mathbb{C}[x,v]$, then $L[t]=K[X,Y,Z]$. The main idea is to study d/dt as a K-derivation of $K[X,Y,Z]$.