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Modules of covariants in modular invariant theory

Let the finite group G act linearly on two vector spaces V and M over an arbitrary field k. The invariant ring $k[V]^G$ has as module $(k[V] \otimes_k M)^G$, called the module of covariants of type M. We study some of the properties of these modules, extending classical results form the non-modular situation to the general. As application, we get results on extensions of invariant rings.

For example, we give a generalization of a theorem of Serre's, saying that if $k[V]$ is free over $k[V]^G$, then G is generated by reflections. Put W for the (normal) subgroup of reflections in G on V and suppose $H < G$ is a subgroup such that $k[V]^G \subset k[V]^H$ is a free extension. Then $G = WH$, i.e., G is generated by H and the reflections in G. Furthermore, multiplication induces an isomorphism

$$k[V]^W \otimes_{k[V]^G} k[V]^H \to k[V]^{H \cap W},$$

hence $k[V]^W \subset k[V]^{W \cap H}$ is also a free extension and $k[V]^{H \cap W}$ is generated by W-invariants and H-invariants.

This is a report on joint work with Jianjun Chuai.