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Modular forms and cycles on ball quotients

This lecture will give an overview of some relations between modular forms, especially their Fourier coefficients, and the geometry and arithmetic of ball quotients $Y = \Gamma\backslash D$, where D is the unit ball in $\mathbb{C}^n$ and Γ is an arithmetic subgroup of the unitary group $\mathrm{U}(n,1)$. Such quotients Y are quasi-projective varieties with many special algebraic cycles arising as quotients $Z_x = \Gamma_x\backslash D_x$ of sub-balls $D_x \subset D$.

By results of old joint work with John Millson, the generating series for the cohomology classes determined by suitable collections of such cycles are modular forms for unitary groups $\mathrm{UU}(r,r)$. This gives an analogue of the seminal results of Hirzebruch–Zagier on generating series for divisors on Hilbert modular varieties.

Recently, in joint work with Michael Rapoport, we define arithmetic analogues of the cycles Z_x by utilizing the fact that the ball quotients Y can be viewed as the complex points of moduli schemes for abelian varieties. We conjecture that the classes in arithmetic Chow groups defined by suitable collections of such cycles are, again, the Fourier coefficients of certain modular forms.

I will discuss some evidence for this conjecture, particularly for the case of arithmetic 0-cycles.