Topology, Knots and Related Fields
Topologie, noeuds et sujets relies
(Org: **Michael Boileau** (Toulouse) and/et **Steven Boyer** (UQAM))

HANS BODEN, McMaster University
Casson's invariant and spliced sums

The $\mathrm{SU}(2)$ Casson invariant $\lambda_{\mathrm{SU}(2)}$ is additive under connected sum and more generally under spliced sum. Although the $\mathrm{SU}(3)$ Casson invariant $\lambda_{\mathrm{SU}(3)}$ is not additive under connected sum, additivity can be achieved by subtracting a suitable multiple of $\lambda^2_{\mathrm{SU}(2)}$. It would be natural to expect this same linear combination of $\lambda_{\mathrm{SU}(3)}$ and $\lambda^2_{\mathrm{SU}(2)}$ to be additive for spliced sums, but simple examples show this is not the case. This talk will report on joint work with Ben Himpel on computations of $\lambda_{\mathrm{SU}(3)}$ for spliced sums of torus knots. A key step is to establish a splitting formula for the spectral flow of the odd signature operator. This approach has been applied to give a formula for $\lambda_{\mathrm{SU}(3)}$ for spliced sums along $(2,p)$ and $(2,q)$ torus knots. To handle splicings along other torus knots requires another key step, namely using perturbations to resolve singularities in the moduli space.

This is the subject of ongoing work with Chris Herald and Ben Himpel.

RYAN BUDNEY, Mathematics and Statistics, University of Victoria, Victoria, BC, Canada V8W 3P4
3-manifolds in the 4-sphere

This project was born out of the observation that there's not many papers in the literature that have results on which 3-manifolds admit smooth embeddings into the 4-sphere. For some families of 3-manifolds there are strong results, namely manifolds that fibre over a surface, and also for homology spheres. But for many other families of 3-manifolds there are few results on "both sides": namely, lack of computable embedding obstructions, and lack of useful embedding constructions. I have been working through the Burton/Martelli/Matveev/Petronio census of prime 3-manifolds that admit (semi-simplicial) triangulations with 11 or less tetrahedra. There's 13400 closed orientable manifolds in the list. As I write this abstract, 19 of the 13400 manifolds in the list are known to embed, and there's only 94 manifolds in the list for which it is not known if they embed (and only 34 of the 94 have trivial JSJ-decompositions). I will briefly describe the methods to compile this data: the computer software used, obstructions to embeddings and how they are computed, and techniques to construct embeddings. I hope to highlight some of the pleasant aspects of the data, including patterns in the data that could give reasonable conjectures for some families of 3-manifolds.

OLIVIER COLLIN, UQAM
Floer spheres, small cyclic branched covers and the Kontsevich integral

Finding Floer spheres—irreducible homology spheres with trivial instanton Floer homology $HF_*(Y)$—is an intriguingly difficult problem, stated as Problem 3.106 in Kirby's updated problem list. The Casson invariant $\lambda(Y)$ can be regarded as a first obstruction to their existence. In this lecture, we are interested in the case of cyclic branched covers along knots, where the covering action $\tau\colon Y \to Y$ can be exploited to detect non-trivial actions on Floer homology or essential surfaces in Y. In particular, the 2-loop part of the Kontsevich integral of a knot can then be interpreted as a second obstruction to the existence of Floer spheres in the class of cyclic branched covers along knots. As an application, we obtain non-vanishing results for the Floer homology of various classes of cyclic branched covers which are small 3-manifolds.

PIERRE DERBEZ, Université Aix Marseille I
Maximal volume of graph manifolds

We first deal with the following question: given a closed orientable 3-manifold N, is there a Lie group G such that the maximal volume $\mathrm{vol}_G(N)$ of all representations of $pi_1 N$ into G does not vanish? When N is a geometric 3-manifold the answer is well known but the question is open for non-geometric manifolds. In the first part of the talk we define a large family of closed non-geometric graph manifolds whose maximal volume is virtually non-zero.

In the second part we use this family to study the following problem of M. Gromov: for which closed orientable 3-manifolds N the set $D(M, N)$ of all possible degree of maps from M to N is finite for any closed orientable 3-manifold M? This question is still open when N is a non-geometric graph manifold. We show that in this case the set $D(M, N)$ is finite for any graph manifold M.

Joint work with Shicheng Wang.

PAOLO GHIGGINI, CalTech

FRANÇOIS GUERITAUD, Université de Paris 11, Orsay, France
Which arborescent links are hyperbolic?

Arborescent links in the 3-sphere form an interesting class with several topological characterizations: for example, these are the links whose associated double branched covers are graph manifolds. It has been known since unpublished work of Bonahon and Siebenmann which arborescent links are hyperbolic. I will outline a new, self-contained proof of this result, using a generalization of the notion of angle structures for a triangulation, or "how to pretend you are doing hyperbolic geometry when you are really doing linear algebra".

Joint work with David Futer.

CYRIL LECUIRE, Université Paul Sabatier, Toulouse, France
Deformation space of Kleinian groups

We study the topology of the analogous of the Teichmüller space for 3-dimensional hyperbolic manifolds with boundary. Consider the set of faithful discrete representations (in the group of isometries of the 3-dimensional hyperbolic space) of a Kleinian group. Several topologies have been defined on this space and they give it different shapes. We will discuss some recent and less-recent works that help us understand these shapes.

CHRISTINE LESCOP, Institut Fourier, Université de Grenoble
Links between Seifert surfaces and finite type invariants of 3-manifolds

In [math.GT/0703347], I proved some surgery formulae for finite type invariants of homology 3-spheres. These surgery formulae that involve link Seifert surfaces specify the relationships discovered by many mathematicians—including Ohtsuki, Goussarov, Habiro, Garoufalidis and Polyak—between various filtrations of the rational vector space freely generated by homology 3-spheres.

I wish to explain how these surgery formulae can be easily guessed from the Kontsevich–Kuperberg–Thurston definition of a universal finite type invariant for homology 3-spheres in terms of configuration space integrals. I shall begin the talk with the simplest case of the degree one Casson invariant.

MELISSA MACASIEB, The University of British Columbia
Character varieties of a family of 2-bridge knot complements

To every hyperbolic finite volume 3-manifold M, one can associate a pair of related algebraic varieties $X(M)$ and $Y(M)$, the $\mathrm{SL}_2(\mathbb{C})$- and $\mathrm{PSL}_2(\mathbb{C})$-character varieties of M. These varieties carry much topological information about M, but are in general difficult to compute. If M has one cusp, then both these varieties have dimension one. In this talk, I will also show how to obtain explicit equations for the character varieties associated to a family of hyperbolic two-bridge knots $K(m,n)$ and discuss consequences of these results related to the existence of integral points on these curves.

This is joint work with Kate Petersen and Ronald van Luijk.

SYLVAIN MAILLOT, Université Louis Pasteur, Strasbourg, France
Ricci flow with surgery on open 3-manifolds and positive scalar curvature

In his proof of the Poincaré Conjecture, G. Perelman constructs an object called Ricci flow with surgery on every closed, orientable 3-manifold. I will describe a variant of this construction, which works in the more general setting of complete 3-manifolds of bounded geometry. I will outline applications to the topological classification of complete 3-manifolds with positive scalar curvature and to the Poincaré Conjecture.

This is joint work with Laurent Bessières and Gérard Besson.

JULIEN MARCHÉ, Université Pierre et Marie Curie, 175 rue du Chevaleret, 75013 Paris, France
Kauffman algebras and geometry of the character variety of a surface in $\mathrm{SU}(2)$

In this talk, we will present some relations between the Kauffman algebra of a surface (that is a formal product on a space of curves on a surface depending on a parameter) and the character variety of the surface in $\mathrm{SU}(2)$. These relations are not completely understood and comprise a well-known isomorphism between the Kauffman algebra with parameter -1 and the algebra of regular functions on the chararacter variety. But the Kaufmann algebra at roots of unity is also a key ingredient for the construction of topological field theories. We will give another relation at parameter $-i$ which makes use of the symplectic structure and the Chern–Simons line bundle over the character variety.

LUISA PAOLUZZI, Université de Bourgogne, IMB, BP 47870, 9 av. Alain Savary, 21078 Dijon cedex, France
Periodic diffeomorphisms with trivial quotient and a characterisation of the 3-sphere

The 3-sphere is the only integral homology sphere admitting four periodic diffeomorphisms with pairwise distinct odd prime orders and trivial quotient, *i.e.*, with space of orbits homeomorphic to $\mathbf{S}^3$. My talk will be devoted to the proof of this characterization of $\mathbf{S}^3$. I shall also discuss some applications to knots and their cyclic branched covers and some possible generalisations of the main result.

This is joint work with M. Boileau (Toulouse) and B. Zimmermann (Trieste).

DALE ROLFSEN, University of British Columbia, Vancouver, Canada V6T 1Z2
Braid groups and the space of orderings

P. Dehornoy showed in the 1990's that the braid groups are left-orderable—that is, its elements can be totally ordered by an ordering which is preserved by left multiplication. More recently, Sikora introduced a natural topology on the set of all left-orderings of a group G, forming a space called $\mathrm{LO}(G)$, which is compact and totally disconnected. He showed that in many cases, $\mathrm{LO}(G)$ is homeomorphic with the Cantor set. It turns out that the braid groups B_n have an interesting space of orderings, in that $\mathrm{LO}(B_n)$ is NOT a Cantor set. I will discuss the isolated orderings of B_n discovered by Dubrovin and Dubrovina, and show that the Dehornoy ordering is not isolated in $\mathrm{LO}(B_n)$. Moreover, I will show that $\mathrm{LO}(B_n)$ contains a Cantor set of orderings, all of which well-order the monoid of Garside positive braids.

This is joint work with Adam Clay.

GENEVIEVE WALSH, Tufts University Medford, MA
Commensurability of knot complements

We discuss the commensurability of hyperbolic knot complements in S^3. It is conjectured that there are at most three hyperbolic knot complements in a given commensurability class. We show that in the case of no hidden symmetries, this conjecture is true. It follows from the proof that a non-fibered hyperbolic knot complement which does not admit hidden symmetries is not commensurable with a fibered knot complement.

This is joint work with M. Boileau and S. Boyer.

BERT WIEST, Université de Rennes 1
Quasigeodesics in the pants complex and the conjugacy problem in braid groups

I shall present some ideas on how to estimate distances in the pants complex, or equivalently in the Teichmueller space with the Weil–Peterson metric, of an n times punctured sphere. Then I want to explain how such estimates could be used to solve the conjugacy problem in the 4 strand braid group in polynomial time. This is work in progress.

DANI WISE, McGill University, Montreal
The Structure of Haken Hyperbolic 3-manifold Groups

A *graph group* is a group whose presentation is obtained by removing some of the relators from the usual presentation for a free-abelian group.

Let M be a hyperbolic 3-manifold with a geometrically finite incompressible surface. I will describe ongoing research towards proving that $\pi_1 M$ virtually embeds in a graph group.

This substantially illuminates the subgroup structure of $\pi_1 M$. In particular, it leads to the subgroup separability of $\pi_1 M$ as well as to a finite cover of M that fibers over the circle.