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Pattern-Avoiding Permutations: A Probabilist's View

A pattern of length k is a permutation $(a[1],\ldots,a[k])$ of $\{1,\ldots,k\}$. This pattern is said to be contained in a permutation of $\{1,\ldots,N\}$ (for $N>k$) if there is a subsequence of k elements of the (long) permutation that appears in the same relative order as the pattern. (E.g. the pattern (132) is contained in the permutation (6425713) because the permutation contains the subsequence (273).) For a given pattern P, let $A_N[P]$ be the number of permutations of $\{1,\ldots,N\}$ that do not contain P. Combinatorialists have proven that $A_N[P]$ grows exponentially in N (rather than factorially), but little is known about the numerical value of the exponential growth rate. For example, which patterns of length 5 are easiest/hardest to avoid?

After introducing the background, I shall describe some Monte Carlo investigations into this and related problems. The design and implementation of the Monte Carlo method raise some interesting mathematical problems. The results of the Monte Carlo lead to some new conjectures, including a description of what a "typical" 4231-avoiding permutation looks like. Finally, I shall outline some rigorous progress on this last problem. The Monte Carlo work was done by Hailong Liu as an NSERC Undergraduate Student Research Awardee.