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On the stability of ground states and the singularity formation for the gravitational Vlasov–Poisson system

I will consider the three dimensional relativistic gravitational Vlasov–Poisson system

$$\begin{cases} \partial_t f + \frac{v}{\sqrt{1+|v|^2/c^2}} \cdot \nabla_x f - \nabla \phi_f \cdot \nabla_v f = 0, \\ f(t=0, x, v) = f_0(x, v) \geq 0, (t, x, v) \in \mathbb{R}_+ \times \mathbb{R}^3 \times \mathbb{R}^3 \end{cases} \quad (1)$$

where ϕ_f is the Poisson gravitational field:

$$\phi_f(x) = -\frac{1}{4\pi} \frac{1}{|x|} \star \rho_f, \quad \rho_f(x) = \int_{\mathbb{R}^3} f(x, v) dv, \quad (2)$$

and $c \in]0, +\infty]$ is the 'light speed'. The value $c = +\infty$ recovers the classical Vlasov–Poisson system which is a nonlinear transport equation describing the mechanical state of a stellar system subject to its own gravity. A well known fact is that smooth solutions to the classical system are global in time while the relativistic system $c < +\infty$ may develop finite time blow up singularities. I will first discuss both in the classical and relativistic settings the question of the existence and stability of ground states stationary solutions for these systems. I will then focus onto the singularity formation problem in the relativistic case and prove the existence of a *stable* self similar blow up dynamics corresponding to a concentration phenomenon for the distribution function.

This is a joint program with Mohammed Lemou (IMT, Toulouse) and Florian Mehats (IRMAR, Rennes).