BOUALEM KHOUIDER, University of Victoria
An inviscid regularization for the SQG equations

One outstanding open question of fundamental interest in fluid mechanics concerns the existence of long-time smooth solutions for the 3D Euler equations. It is argued in the literature that the surface quasi-geostrophic (SQG) equations bear some resemblance with the 3D Euler vorticity equations and as such they provide an interesting model for studying long time behaviour of solutions in a context similar to Euler equations. As an alternative to previous interesting approaches by others, we propose here an inviscid regularization for the SQG equations to facilitate the analysis. The new regularization yields a necessary and sufficient condition, satisfied by the regularized solution, when a regularization parameter tends to zero, for the solution of the original SQG equations to develop a singularity in finite time. As opposed to the commonly used viscous regularization, the inviscid equations derived here conserve a modified energy.

Therefore, the new regularization provides an attractive numerical procedure for finite time blow up testing. In particular, we prove that, if the initial condition is smooth, then the regularized solution remains as smooth as the initial data for all times. Some numerical tests will be presented as well.

Joint work with Edriss Titi.