Financial Mathematics
Mathématiques financières
(Org: **Tom Salisbury** (York))

WAEL BAHSOUN, University of Manchester
Evolutionary models of financial markets

The idea of this direction of work is to apply evolutionary dynamics (mutation and selection) to the analysis of the long-run performance of financial trading strategies. A stock market is understood as a heterogeneous population of frequently interacting investment strategies (portfolio rules) in competition for market capital. The aim of the work is to build a "Darwinian theory" of portfolio selection. The paper reviews recent results pertaining to models of financial evolutionary dynamics based on the theory of random dynamical systems. The main focus is on the version of the model where investors adopt fixed-mix portfolio rules prescribing to invest their wealth in the assets according to constant, time-independent proportions. It is shown that, in this setting, the Shannon–Kelly–Breiman strategy of "betting one's beliefs" is dominant: those traders who adopt it eventually gather total market wealth. Variants of this result dealing with more general, not necessarily fixed-mix, portfolio rules are discussed.

This is joint work with Igor Evstigneev (University of Manchester).

MATT DAVISON, University of Western Ontario
Optimal Investment and Consumption under Habit Formation

In the standard Merton formulation of optimal investment and consumption, the integrated discounted lifetime utility of consumption together with the discounted future bequest, are optimized. In this standard formulation the time t utility of consumption depends only on the amount consumed at that time. However, it is both theoretically and empirically reasonable to suppose that the utility of consumption depends on past individual consumption history. Economists term this 'habit formation'. In this paper we construct a simple mathematical description of this habit formation and derive Merton style nonlinear partial differential equations (PDEs) having an additional 'spatial' variable for the resulting investment-consumption problem. We present numerical solutions for the resulting PDE and draw insights from these solutions.

This is joint work with Roman Naryshkin (Western Ontario)

TOM HURD, McMaster University
Credit Risk using Time Changed Brownian Motions

Time changed Brownian motions such as the variance gamma model have been very popular for modelling equities, but less applied in the credit risk domain. The reason for their late adoption despite their versatility may be the intractable nature of the first passage problem in these models. In this talk I show how this difficulty can be circumvented. Thus freed, we are able to investigate some consequences of adding jumps to firm value (structural) models. We then move to multivariate credit models and consider the implications of introducing dynamic default correlations through time change.

FELIX KAN, University of Western Ontario
Pricing American Options by Simulation: Bias Reduction on Modified Least-Squares Monte Carlo

Existing Monte Carlo algorithms for pricing American options generate estimators that are consistent but biased. Whitehead, Davison and Reesor (2007) introduce a general bias reduction technique for pricing American options by Monte Carlo methods based on large sample theory that corrects the estimators from the stochastic tree and mesh techniques. We apply this

technique to a modified version of least-squares Monte Carlo method of Longstaff and Schwartz (2001) using well-known results about the large sample properties of least-squares estimators. We derive an expression for bias-corrected estimators. Numerical results show the effectiveness of this technique.

This is joint work with Mark Reesor (Western Ontario).

ALEXEY KUZNETSOV, University of New Brunswick, Saint John

Pricing barrier options in the Variance-Gamma model

One of the main criteria for evaluating a pricing model is its ability to produce analytical formulas for the prices of benchmark options. Variance-Gamma model has already proved to be a reasonable generalization of the Black–Scholes model: it can provide a better fit to the market data and at the same time the prices of European options can be computed in (almost) closed form. Unfortunately the picture is completely different for the barrier options: the payoffs of these weakly path-dependent derivatives depend on the first passage time of the stock, and this random variable is very hard to control in the case of infinite-activity Levy jump processes. In this talk we will present a new, semi-analytical method for computing distribution of the first passage time. In our approach we use the construction of the VG process as a time changed Brownian motion to extract additional information about the first passage time. We will present numerical results and a comparison of our method to PIDE and Wiener–Hopf methods.

ALEXANDER MELNIKOV, University of Alberta

Financial Market Modeling via Telegraph Processes

The talk develops a class of financial market models based on generalized telegraph processes: Markov random flows with alternating velocities and jumps occurring when the velocities are switching. While such markets may admit arbitrage opportunities, the model under consideration is arbitrage-free and complete if directions of jumps in stock prices are in a certain correspondence with stock prices velocities and interest rates behavior. An analog of the Black–Scholes fundamental differential equation is derived, but in contrast with the Black–Scholes model, this equation is hyperbolic. Explicit formulas for prices of European options are obtained using perfect and quantile hedging.

The talk is based on a joint paper with N. Ratanov, *On financial markets based on telegraph processes*, Stochastics **80**(2008), 247–268.

VLADIMIR SURKOV, University of Toronto

FFT-Based Option Pricing under Mean-Reverting Jump Diffusion

Energy commodities, such as oil, gas and electricity, exhibit high volatilities, have sudden price jumps and tend to revert to a long run equilibrium. We develop an FFT-based method for valuing path-dependent contingent claims written on mean-reverting processes with jumps. The method is efficient as European options can be priced using a single time-step to obtain option values for a range of spot prices. Furthermore, Bermudan options do not require time-stepping between monitoring dates and the method can be readily extended to the multi-asset framework. We carry out several pricing experiments on European, American styled swing and two-asset spread options.

ANATOLY SWISHCHUK, University of Calgary

Pricing of Variance Swaps for Stochastic Volatilities with Delay and Jumps

The valuation of the variance swaps for stochastic volatility with delay and jumps is discussed in this talk. We provide some analytical closed forms for the expectation of the realized variance for the stochastic volatility with delay and jumps. The jump part in our model is finally represented by a general version of compound Poisson processes. As applications of our analytical solutions, a numerical example using S&P60 Canada Index (1998–2002) is then provided to price variance swaps with delay

and jumps. Finally, we find that this model not only keeps some good features of the previous model without jumps but is also easy and quick to implement.

This is a joint work with Li Xu (University of Calgary).