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Shape Optimization and Free Boundary Problems in Fluid Mechanics

In this presentation we discuss a technique for the computational solution of some classes of free boundary problems arising in fluid mechanics. In such problems the shape of the domain where the governing PDEs are stated in unknown and must be determined as a part of the solution of the problem. As examples we consider the following two model problems:

(i) the *direct* Stefan problem where one seeks to find the phase-change interface between the liquid and solid phases in the presence of a contact line, and

(ii) the *inverse* problem of Vortex Design where one seeks to determine the boundary conditions for the incompressible Euler equations, so that the vortex region in the flow will have a prescribed shape.

We show that both of these problems can be efficiently solved using methods of the PDE-constrained optimization, where the unknown shape of the domain boundary is sought as a minimizer of a suitable cost functional. We demonstrate how the optimality conditions for such problems are derived using the *shape calculus* which allows one to differentiate solutions of PDEs with respect to the geometry of the domain. Such optimal domain shapes can be found computationally using a gradient-based algorithm, where the gradient of the cost functional can be conveniently computed by solving an adjoint PDE system. In addition to discussing the mathematical foundations of this method we will also present computational examples illustrating its performance on the Stefan and Vortex Design problems.

Joint work with Oleg Volkov.