
Complex Dynamical Systems
Systèmes dynamiques complexes
(Org: **Xavier Buff** (Toulouse), **Tan Lei** (Cergy-Pontoise) and/et **Misha Lyubich** (Toronto))

MAGNUS ASPENBERG, Institute of Mathematics, IMPAN, Warsaw
Collet–Eckmann and Misiurewicz maps

A well studied type of non-uniformly expanding maps are the Collet–Eckmann maps (CE), introduced in the 1980s by P. Collet and J-P. Eckmann. These maps has positive Lebesgue measure in the space \mathcal{R}_d of rational functions for any degree $d \geq 2$, which was shown in my thesis in 2004. A special type of CE-maps are the so-called Misiurewicz maps, which have no parabolic cycles and the critical set on the Julia set is non-recurrent (they are also assumed to be non-hyperbolic). These maps have Lebesgue measure zero but full Hausdorff dimension in \mathcal{R}_d for any $d \geq 2$. Moreover, every Misiurewicz map can be approximated by a hyperbolic map (answering a conjecture by Herman). Some results of this type also extends to semi-hyperbolic maps, which do not have any parabolic cycles and for which every critical point on the Julia set is non-recurrent (instead of the whole critical set).

FRANÇOIS BERTELOOT, Université Paul Sabatier, Toulouse, France
Equidistribution of polynomials having a neutral cycle

Within the family \mathcal{P}_d of degree d polynomials, we investigate the distribution of the hypersurfaces $\text{Per}_n(w)$ which consist of polynomials possessing a n -cycle of multiplier w . Using properties of Lyapunov exponents, we show that the sequence of weighted current of integration $d^{-n}[\text{Per}_n(e^{i\theta})]$ is converging to the bifurcation current T_{bif} for any $(e^{i\theta}) \neq 1$.

ILIA BINDER, University of Toronto
Harmonic measure and Walk on Spheres algorithm

We investigate the rate of convergence of the Walk on Spheres algorithm, an efficient method for computing harmonic measure.

XAVIER BUFF, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse, France
Siegel disks of cubic polynomials

Given λ in the unit disk, we consider the family of cubic polynomials $P_{\lambda,a}(z) = \lambda(z + az^2 + z^3)$. There is a unique power series $\phi_{\lambda,a}(z) = z + o(z)$ such that $\phi_{\lambda,a}(\lambda z) = P_{\lambda,a}(\phi_{\lambda,a}(z))$. We let $r_\lambda(a)$ be the radius of convergence of the series $\phi_{\lambda,a}$. The function $u_\lambda = -\log r_\lambda$ is continuous and subharmonic. We study the behavior of the functions u_λ and the measures $\mu_\lambda := \Delta u_\lambda$ as $|\lambda| \rightarrow 1$.

Joint work with Arnaud Chéritat and Carsten Petersen.

SERGE CANTAT, CNRS, Université de Rennes 1, France
Dynamics on Character Varieties and Quasi-Fuchsian Representations

I will survey recent results concerning the dynamics of the mapping class group of the four punctured sphere \mathbb{S}_4^2 on the character variety, i.e., on the space of conjugacy classes of representations of the fundamental group of \mathbb{S}_4^2 into $\text{SL}_2(\mathbb{C})$. Using tools from holomorphic dynamics one can study orbits of this action and get a new viewpoint on the set of quasi-fuchsian representations.

LAURA DEMARCO, University of Illinois at Chicago, Chicago, IL, USA

Polynomial escape combinatorics

In dynamical space \mathbb{C} , collapsing each connected component of a level set of the escape-rate function to a point yields a tree. I will describe a parameter-space analog: if you collapse each connected component of a fiber of the critical escape-rate map, you obtain a space of “augmented trees”.

BOB DEVANEY, Boston

JEREMY KAHN, Stony Brook University, Stony Brook, NY

Bounds for bounded-primitive type

We say that an infinitely renormalizable quadratic polynomial $z \mapsto z^2 + c$ has *bounded-primitive* type if the ratio between consecutive primitive renormalization times is bounded. We prove the *a priori* bounds for bounded-primitive type maps, which establishes the local connectivity of the Mandelbrot set at the corresponding parameter values.

VOLKER MAYER, Université Lille I

Ergodic properties of some sub-hyperbolic meromorphic functions

We present joint work with M. Urbanski on ergodic properties of sub-hyperbolic meromorphic functions with polynomial Schwarzian derivative and, in particular, establish the existence, ergodicity and unicity of a conformal measure with minimal exponent h together with a Bowen’s formula showing that this number h equals the Hausdorff dimension of the Julia set of the function. So, as expected, the theory of these sub-hyperbolic functions seems to behave like the theory of hyperbolic functions. However, it turns out that there is a significant difference: there always exists an invariant measure which is absolutely continuous with respect to the above conformal measure but we will see that, in many cases, this measure is only σ -finite.

KEVIN PILGRIM, Indiana University, Dept. of Mathematics, Rawles Hall, Bloomington, IN 47405, USA

On Thurston’s pullback map

We discuss properties of the so-called Thurston pullback map on Teichmueller space associated to a postcritically finite branched covering of the two-sphere to itself.

This is joint work with X. Buff, A. Epstein, and S. Koch.

ROLAND ROEDER, University of Toronto, Toronto, ON M5S 2E4

Complex dynamics for the Lee–Yang zeros

In a classical work, Yang and Lee proved that zeros of certain polynomials (partition functions of Ising models) always lie on the unit circle. Distribution of these zeros control phase transitions in the model. We study this distribution for a special “Migdal–Kadanoff hierarchical lattice”. In this case, it can be described in terms of the dynamics of an explicit rational function in two variables. We show that the Yang–Lee zeros are organized in a transverse measure for a dynamical foliation on an invariant cylinder. From the complex point of view, they get interpreted as slices of a dynamical $(1, 1)$ -current on the projective space.

This is a joint work with Pavel Bleher and Mikhail Lyubich.

PASCALE ROESCH, Institut de Mathematiques de Toulouse, 118 route de Narbonne, F-31062 Toulouse Cedex 9, France
The boundary of polynomial Fatou components

I will give an idea of the proof of the following result obtained with Yin Yongcheng.

Theorem *For a polynomial, any bounded Fatou component, which is not a Siegel disk, has a boundary which is a Jordan curve.*

VLADLEN TIMORIN, Jacobs University Bremen
Regluing of holomorphic functions

Regluing is a topological operation that helps to construct topological models for rational functions on the boundaries of certain hyperbolic components. It also has a holomorphic interpretation, with the flavor of infinite dimensional Thurston–Teichmüller theory.

MISHA YAMPOLSKY, University of Toronto, Canada
A survey of computability results for Julia sets

A compact set in the plane is computable if there is an algorithm to draw it on a computer screen with an arbitrary resolution. Together with Mark Braverman we have investigated computability questions for Julia sets. I will survey the results, some of which were unexpected. In particular, even when the parameters of the rational map are computable, it is possible that its Julia set is non-computable.