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Ring solutions in R^N and smoke-ring (vortex) solutions in R^3 for Gierer–Meinhardt Model

We consider the classical Gierer–Meinhardt Model in N dimensions,

$$\varepsilon^2 \Delta u - u + \frac{u^p}{v^q} = 0, \quad \Delta v - v + \frac{u^m}{v^s} = 0$$

where ε is assumed to be small.

A ring-type solution in $\mathbb{R}^N$ is a solution that concentrates on the surface of an N-sphere as $\varepsilon \to 0$. On the other hand, a smoke-ring or vortex solution in $\mathbb{R}^3$ is a solution that concentrates on the perimeter of a two-dimensional circle.

For ring solutions, assume

$$0 < \frac{p-1}{q} < a_\infty \text{ if } N = 2, \quad \text{and} \quad 0 < \frac{p-1}{q} < 1 \text{ if } N \geq 3$$

where $a_\infty > 1$ whose numerical value is $a_\infty = 1.06119$. We prove that there exists a unique $R_a > 0$ such that for $R \in (R_a, +\infty]$, there is a ring-type solution inside the ball of radius R ($R = +\infty$ corresponds to $\mathbb{R}^N$ case), that concentrates on the surface of a ball of radius $0 < r_0 < R$. Moreover depending on parameter values, there are either exactly one or two choices for r_0.

For smoke-ring solutions, we study the case when the domain is all of $\mathbb{R}^3$. We then show that a smoke-ring solution concentrates on a circle whose radius is precisely $r_0 = 0.43385$.

The analysis of ring solutions relies heavily on manipulation of Bessel functions. The analysis for smoke-ring solutions involves a deep expansion of a certain singular integral.

This is a joint work with Juncheng Wei (rings) and with Xiaofeng Ren (smoke-rings).