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Random walks in symmetric random environments

This talk will describe recent progress in the study of symmetric (or time reversible) random walks in random environments. There is a close connection with the homogenization of PDE. Consider the initial value problem

$$\frac{\partial u}{\partial t} = \sum_{ij} \frac{\partial}{\partial x_i} a_{ij}(x/\varepsilon) \frac{\partial}{\partial x_j} u_\varepsilon(t,x), \tag{1}$$

where $x \in \mathbb{R}^d$, $u_\varepsilon(0,x) = v_0(x)$, and $a(x) = \big(a_{ij}(x)\big)$ is symmetric. This equation describes diffusion in an irregular medium with fluctuations at length scale ε. The theory of homogenization is most developed in the case when $a(\cdot)$ is uniformly elliptic and periodic. Significant progress has been made in the relaxation of the hypothesis of periodicity, for example by making a a stationary random field. However, if one allows a to be zero, the set $Z = \{x : a(x) = 0\}$ acts as a barrier to diffusion, and one needs to consider carefully the structure of the set $C = \mathbb{R}^d - Z$ on which diffusion can occur.

I will discuss a discrete version of this problem. Here $\mathbb{R}^d$ is replaced by the lattice $\varepsilon\mathbb{Z}^d$, and the set C by the unique unbounded connected component of a supercritical percolation process on $\varepsilon\mathbb{Z}^d$. I will discuss Gaussian bounds, homogenization, Harnack inequalities and Green's functions in this setting. The differential inequalities that Nash introduced in his 1958 paper are particularly well suited to this problem.