Considerable experimental and theoretical effort has been devoted to obtaining the asymptotic scaling of the enhanced bulk heat transport in turbulent Rayleigh–Bénard convection, measured by the Nusselt number, in terms of the temperature drop across the fluid, given by the Rayleigh number; however, the usual assumption of fixed temperature across the fluid is mathematically and experimentally inadequate.

We formulate a variational bounding principle to obtain rigorous theoretical estimates for the Nusselt number as a function of the Rayleigh number in finite Prandtl number turbulent convection. We are able to treat a full range of thermal boundary conditions between the fixed temperature and fixed flux extremes in a uniform formulation, and show that the usual fixed temperature assumption is a singular limit of the full problem. We also obtain analytical bounds in the physically realistic case of a fluid bounded by conductive plates, and discuss some generalizations.