XUEQING CHEN, University of Wisconsin–Whitewater
Quivers, Ringel-Hall Algebras and Lie Theory

Let $g = g(C)$ be the Kac–Moody Lie algebra associated to a Cartan matrix C and $\mathbf{U} = \mathbf{U}_v(g)$ its quantum group. A key feature in quantum groups is the presence of several natural bases (like the PBW-basis and the canonical basis). There are different approaches to the construction of the canonical basis: algebraic approach, geometric approach and Ringel–Hall algebra approach. In this talk, we start by recalling the basic theory of quivers and Ringel–Hall algebras, paying special attention to Gabriel's Theorem and Ringel–Green's work on the realization of quantum groups and Lie algebras by using Hall algebras of finite dimensional associative algebras. We will then recall algebraic and Ringel–Hall algebra approaches to a PBW basis and a canonical basis of $\mathbf{U}$ when C is of finite or affine type. Meanwhile, the root vectors in Ringel–Hall algebras will be discussed. Finally, we shall go on to discuss some of the many further developments and applications of the theory.