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Bit-Thrifty Algorithms for Facility Location

Let S be a sequence of n points whose coordinates are given initially only to some limited precision, with additional precision available at some additional cost per bit. Suppose that we wish to determine some property P of S: for example, what is the 1-centre of S, or (even more simply) does one distinguished point of S lie in the interior of the convex hull of the rest. For some problem instances, many bits of all (or most) of the points need to be determined in order to establish P; for other instances P can be established with the knowledge of only a relatively small (total) number of bits. In general, we would like to design algorithms whose cost (number of bits examined) adapts to the intrinsic cost (shortest computation or "proof" of the property P) for a given instance.

This talk describes some results concerning a few very basic questions of this type, focusing primarily on facility-location type questions for point sets in low dimensions. In many cases we are able to define a notion of intrinsic cost $c(I)$ of an instance (input set) I and show that

(i) any algorithm can be forced to examine $c(I)$ bits in the worst case (over all presentations of I as an input sequence), even if I is known,

(ii) there is an algorithm that examines only $O\big(c(I)\lg c(I)\big)$ bits in the worst case, even if I is not known, and

(iii) any algorithm can be forced to examine $O\big(c(I)\lg c(I)\big)$ bits in the worst case (over all presentations of inputs with intrinsic cost $c(I)$), even if $c(I)$ is known.

Based on joint work with Raimund Seidel.