Nonlinear Methods in Computational Mathematics
Méthodes nonlinéaires en mathématiques computationnelles

(Org: **Kirill Kopotun** (Manitoba))

A. BASS BAGAYOGO, University College of Saint Boniface

Hybrid Octree Grid Based Method and Application to an Aircraft Geometry

Geometry modelling and grid generation over complex objects is one of the important and essential aspect in Computational Fluid Dynamics. As the complexity grows with size, it becomes difficult to visualize and modify the grids. In this talk I will present and hybrid grid generate by using and Octree/Quadtree based method, the emphasis will be on the rapid acquisition of the geometry, the special design data structures, and some aspects related to the intersection algorithms.

FENG DAI, University of Alberta, Edmonton, Alberta

An inequality on m-term approximation by ultraspherical polynomials and its applications

In this talk, I shall show a useful inequality on m-term approximation by ultraspherical polynomials on $[-1,1]$. As an application, I shall show how to use this inequality to construct a sequence of polynomials ψ_j, $j=1,2,\dots$ with the following properties:

(i) $\psi_j \in \operatorname{span}\{P^\lambda_{2^{j-1}+1}, P^\lambda_{2^{j-1}+2}, \dots, P^\lambda_{2^j}\}$, where P^λ_k denotes the usual ultraspherical polynomial of degree k and index λ on $[-1,1]$.

(ii) $\|\psi_j\|_{2,\lambda} \approx \|\psi_j\|_\infty$ with the constant of equivalence independent of j, here $\|\cdot\|_{2,\lambda}$ denotes the L^2 norm on $[-1,1]$ computed with respect to the weight $(1-t^2)^\lambda$.

ZEEV DITZIAN, University of Alberta, Edmonton, Alberta

Sharp Jackson inequalities

Sharp Jackson inequalities are given for some systems of orthogonal functions on various domains.

GERMAN DZYUBENKO, International Mathematical Center of NAS of Ukraine, 01601, Tereschenkivska Str., 3, Kyiv-4, Ukraine

Shape preserving approximation of periodic functions

Let $2s$, $s\in\mathbb{N}$, fixed points y_i $-\pi \le y_{2s} < y_{2s-1} < \cdots < y_1 < \pi$ are given and for the other indexes $i\in\mathbb{Z}$, the points y_i are defined periodically, i.e., by the equality $y_i = y_{i+2s} + 2\pi$, $Y := \{y_i\}_{i\in\mathbb{Z}}$. ¿From the space C of continuous 2π-periodic functions $f\colon \mathbb{R}\to\mathbb{R}$ with the norm $\|f\| := \max_{x\in\mathbb{R}} |f(x)|$, we extract three sets $\Delta^{(q)}(Y)$, $q=0,1,2$, of all functions f which are, respectively, nonnegative/nondecreasing/convex on $[y_1,y_0]$, nonpositive/nonincreasing/concave on $[y_2,y_1]$ and so on. Let

$$E_n^{(q)}(f) := \inf_{T_n\in\mathbb{T}_n\cap\Delta^{(q)}(Y)} \|f-T_n\|, \quad n\in\mathbb{N},$$

where $\mathbb{T}_n$ is the space of trigonometric polynomials of order $\le n-1$.

Theorem 1 *If $f \in \Delta^{(q)}(Y)$ then*

$$E_n^{(q)}(f) \le c(s)\omega_k(f, \pi/n), \quad n \ge N(Y), \quad k = \begin{cases} 2, & \text{if } q = 1, \\ 3, & \text{if } q = 0, 2, \end{cases}$$

where $\omega_k(f,t)$ is the k-th modulus of continuity of f, $c(s)$ and $N(Y)$ are the constants depending only on s and on $\min_{i=1,\dots,2s}\{y_i - y_{i+1}\}$, respectively.

Remark 1 Each of these three estimates is wrong with a greater k. It follows from the Whitney inequality that the constants $c(s)$ and $N(Y)$ can be both replaced simultaneously by $c(Y)$ and 1, respectively. The respective estimates with $c(s)$ and 1 are wrong.

The case $q = 0$ was proved by the author and J. Gilewicz, $q = 1$ by the author and M. G. Pleshakov, $q = 2$ by the pupil of the author V. D. Zalizko.

QIANG GUO, York University
Adaptive wavelet method for aerosol dynamic equation

A new and robust wavelet-based splitting method has been developed to solve the general aerosol equations. The considered models are the nonlinear integro-partial differential equations on time, size and space, which describe different processes of atmospheric aerosols including condensation, ucleation, coagulation, deposition, and sources as well as turbulent mixing.

The proposed method reduces the complex general aerosol dynamic equation to two directional splitting equations. Because there are steeply varying number densities across a size range, an adaptive wavelet strategy is developed to solve the size splitting equation effectively. And further the wavelet method and the finite difference method are alternately used for two directional splitting equations at each time interval.

TOM HOGAN, The Boeing Company, Seattle, WA, USA
Implications of design optimization on geometry generation in aerospace

Airplane design, and vehicle design in general, is evolving. The traditional technique was for an experienced designer/engineer with a real talent for design and a large personal knowledge base to draft a single vehicle in a CAD system; analyze it for pertinent properties (like the lift provided by the wings, the drag of the vehicle, its structural integrity, predicted fuel consumption, etc.); and decide if it meets the market's needs. If it doesn't, which is typical, the next step was essentially to go back to the drawing board to see if it can be tweaked to do so. More recently, the designer may provide a baseline design to which small perturbations can be made. Then an optimization package can try to hone in on an acceptable design... as long as there is one that is nothing more than a minor modification of the baseline.

The next step in this evolution is for the designer to design an entire family of vehicles that depend on a set of parameters, i.e., a bunch of virtual knobs that can be turned to morph the vehicle, allowing for more significant changes so a larger set of vehicles can be studied. In this presentation we show why existing CAD packages are ill-equipped for this new approach, present some of the tools we have developed to address the issues and give a taste of the kinds of mathematics behind these new tools.

YINGKANG HU, Georgia Southern University, Dept. of Math. Sci., Statesboro, GA 30460-8093, USA
Global Optimization using hyperbolic cross points with application in clustering

Erich Novak and Klaus Ritter developed in 1996 a global optimization algorithm that uses hyperbolic cross points (HCPs). We modify this algorithm in many ways to improve its efficiency and developed a local search strategy that results in much better chance to find the global minimizer. The ideas are implemented on the computer for optimization in clustering. The program has been tested extensively with very promising results.

FRANCISCO-JAVIER MUÑOZ-DELGADO, Universidad de Jaen, 23071 Jaen, Spain
Optimal shape perserving linear operators with different types of data

In 1980, H. Berens and R. DeVore (*A characterization of Bernstein polynomials*, in Approximation Theory III, Proc. Conf., Austin, Texas, 1980, 213–219) showed that classical Bernstein operators are the best in certain sense. They proved that if L is a linear operator mapping real functions defined on $[0,1]$ onto polynomial functions of degree less or equal to n, preserving the positivity and the sign of all the derivatives and fixing the linear polynomial, then the eigenvalue corresponding to the polynomial functions of degree two, λ_2, verifies $\lambda_2 \le \frac{n-1}{n}$, and the identity is satisfied only by Bernstein operators.

Now, we consider linear polynomial operators that use certain type of data (values of functions in some points, derivatives, moments, etc.) and we consider the preservation of the sign of only one derivative. For each case, we look for a optimal operator. We show that Bernstein, Bernstein–Kantorovich and Bernstein–Durrmeyer operators are optimals in certain cases. In others, we show new Bernstein-type operators.

BOJAN POPOV, Texas A&M University
L_1 approximations of Hamilton–Jacobi equations

L_1-based minimization method for stationary Hamilton–Jacobi equations

$$H(x,u,Du)=0, \quad x\in\Omega \quad \text{with } u|_{\partial\Omega}=0$$

is developed. The case considered is of a 2D bounded domain with a Lipschitz boundary. The general assumption is that the viscosity solution u of the problem is unique, $u\in W^{1,\infty}(\Omega)$, and the gradient Du is of bounded variation. We approximate the solution to this problem using continuous finite elements and by minimizing the residual in L_1. In the case of a convex (with respect to Du) and uniformly continuous hamiltonian, it is shown that, upon introducing an appropriate entropy, the sequence of approximate solutions based on quasi-uniform shape regular finite element triangulations converges to the unique viscosity solution u. The main features of the method are that it is an arbitrary polynomial order and it does not have any artificial viscosity. The fact that the residual in minimized in L_1 is a key. Numerical examples and possible application of this method to other hyperbolic equations will be discussed.

ANDRIY PRYMAK, CAB 632, Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, AB, T6G 2G1
Ul'yanov-type inequality for bounded convex sets in R^d

For $\Omega\in R^d$, a convex bounded set with non-empty interior, the moduli of smoothness $\omega^r(f,t)_{L_q(\Omega)}$ and the norm $\|f\|_{L_q(\Omega)}$ are estimated by an Ul'yanov-type expression involving $\omega^r(f,t)_{L_p(\Omega)}$ where $0<p<q\le\infty$. The main result for $q<\infty$ is given by

$$\omega^r(f,t)_q \le C\left\{\int_0^t u^{-q\theta}\omega^r(f,u)_p^q\frac{du}{u}\right\}^{1/q}, \quad 0<t\le \operatorname{diam}\Omega, \quad \theta=\frac{d}{p}-\frac{d}{q}.$$

A corresponding estimate of $\|f\|_{L_q(\Omega)}$ is, in fact, an embedding theorem involving Besov spaces with a range of q more general than known today. The power q achieved is optimal.

IGOR SHEVCHUK, National Taras Shevchenko University of Kyiv, Ukraine
Convex and coconvex polynomial approximation in the uniform norm

A survey on the results by K. Kopotun, D. Leviatan, the author, and others.

Let $s \in \mathbb{N}$, $-1 < y_s < \cdots < y_1 < 1$, $Y_s = \{y_i\}_{i=1}^s$, $\Delta^{(2)}(Y_s)$ be the set of continuous on $[-1,1]$ functions, which are convex on $[y_1, 1]$, concave on $[y_2, y_1]$, etc., $\Delta^{(2)}(Y_0)$ be the set of convex continuous on $[-1,1]$ functions, $\|\cdot\|$ be a uniform norm on $[-1,1]$, $\mathbb{P}_n$ be the space of algebraic polynomials of degree less that n, and

$$E_n^{(2)}(f, Y_s) := \inf_{P_n \in \mathbb{P}_n \cap \Delta^{(2)}(Y_s)} \|f - P_n\|$$

be the error of the best uniform coconvex approximation of f.

For $k \in \mathbb{N}$, $r \in \mathbb{N}_0$ and function $f \in C^{(r)} \cap \Delta^{(2)}(Y_s)$ we will discuss the validity of the inequality

$$E_n^{(2)}(f, Y_s) \le \frac{C}{n^r} \omega_k\Big(\frac{1}{n}, f^{(r)}\Big), \quad n \ge N,$$

where ω_k are moduli of smoothness of different types.

PING ZHOU, St. Francis Xavier University

Newton's interpolation formula in study of multivariate functions

We discuss the use of Newton's interpolation formula, i.e., divided differences, in the following studies of multivariate functions:

1. Finite sum representations of some multivariate functions, e.g. the Lauricella function F_D.
2. Explicit constructions of multivariate Padé approximants for pseudo-multivariate functions.
3. Arithmetical results on certain multivariate power series, i.e., the proof of the irrationality and transcendence of some multivariate power series.