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L_1 approximations of Hamilton–Jacobi equations

L_1-based minimization method for stationary Hamilton–Jacobi equations

$$H(x, u, Du) = 0, \quad x \in \Omega \quad \text{with } u|_{\partial\Omega} = 0$$

is developed. The case considered is of a 2D bounded domain with a Lipschitz boundary. The general assumption is that the viscosity solution u of the problem is unique, $u \in W^{1,\infty}(\Omega)$, and the gradient Du is of bounded variation. We approximate the solution to this problem using continuous finite elements and by minimizing the residual in L_1. In the case of a convex (with respect to Du) and uniformly continuous hamiltonian, it is shown that, upon introducing an appropriate entropy, the sequence of approximate solutions based on quasi-uniform shape regular finite element triangulations converges to the unique viscosity solution u. The main features of the method are that it is an arbitrary polynomial order and it does not have any artificial viscosity. The fact that the residual in minimized in L_1 is a key. Numerical examples and possible application of this method to other hyperbolic equations will be discussed.