Model Theory and its Applications
Théorie des modèles et ses applications
(Org: **Bradd Hart** (McMaster), **Thomas Kucera** (Manitoba) and/et **Rahim Moosa** (Waterloo))

JOHN BALDWIN, University of Illinois at Chicago
Notions of Excellence

Excellence is the key property for studying categoricity in infinitary logic. But the notion is only clearly defined and fully worked out for atomic models of a first order theory (thereby coding complete sentences of $L_{\omega_1,\omega}$). We will consider some of the possible ways of extending the definition and some examples.

PAUL BANKSTON, Marquette University, Milwaukee, Wisconsin
Chainability and Unidimensionality from a Model-Theoretic Perspective

On the surface, the textbook definitions of chainability and unidimensionality–in the sense of covering dimension–are quite similar. In this talk we use model theory to explore the assertion that this similarity is only skin deep. In the case of dimension, there is a beautiful theorem of E. Hemmingsen that allows us to give a first-order characterization in terms of the language of lattices. We show that no such characterization is possible for chainability by proving that if κ is any infinite cardinal and $\mathcal{B}$ is an open lattice base for a continuum, then $\mathcal{B}$ is elementarily equivalent to an open lattice base for a continuum X, of weight κ, such that X has a three-set open cover admitting no chain open refinement.

ALF DOLICH, University of Illinois at Chicago, Chicago IL 60607
Independence relations, universality, and SOP_4

In [2] (elaborated upon by Dzamonja and Shelah in [1]), Shelah studies, for a fixed theory T, whether under an appropriate forcing it is consistent that at certain cardinals λ only few models of T of cardinality λ are needed to elementarily embed all models of T of cardinality λ. In [3] Shelah showed that for any simple theory T as well as for various non-simple examples this is the case and that this universality property fails for any theory with the combinatorial property SOP_4. I will discuss work towards developing a uniform framework, based on the existence of independence relations with several weak properties, intended to capture all theories T with the desirable universality property alluded to above.

References

[1] Mirna Dzamonja and Saharon Shelah, *On properties of theories which preclude the existence of universal models.* Ann. Pure Appl. Logic **139**(2006), 280–302.

[2] Saharon Shelah, *The universality spectrum: consistency for more classes.* In: Combinatorics, Bolyai Society Mathematical Studies, 1993, 403–420.

[3] ———, *Toward classifying unstable theories.* Ann. Pure Appl. Logic **80**(1996), 229–255.

ALINA DUCA, University of Manitoba, Winnipeg, MB, R3T 2N2
Description of the injective modules over a principal left and right ideal domain

Over a principal left and right ideal domain R every injective module is a direct sum of indecomposable injective modules. One indecomposable injective is the injective envelope (divisible hull) of the module ${}_RR$ and is isomorphic to the division algebra Q of R. The other indecomposable injective modules are (up to isomorphism) in a one-to-one correspondence with the prime elements of the ring (up to similarity).

Motivated by a classic treatment of O. Ore, I take advantage of the factorization theory in R and investigate the internal structure of an indecomposable injective left module $E \neq Q$. I describe its "layered" structure in terms of its elementary socle series $\left(\operatorname{soc}^{\alpha}(E)\right)_{\alpha}$, a concept which was introduced by I. Herzog as the elementary analogue of the socle series of a module, where the minimality condition on the pp-definable subgroups is used. Since E has the descending chain condition on pp-definable subgroups, the elementary socle series exhausts E. A complete characterization of $\left(\operatorname{soc}^{\alpha}(E)\right)_{\alpha}$ is obtained.

In addition, I will analyze the relationship between the classical socle series of the right module E over the ring $T = \operatorname{End}_R(E)^{\mathrm{op}}$ and the elementary socle series of the left R-module E.

DRAGOS GHIOCA, McMaster University
A dynamical version of the Mordell–Lang Theorem

We prove a dynamical version of the Mordell–Lang conjecture for the affine line. We use methods from number theory and model theory. ¿From number theory we use analytic methods similar to the ones employed by Skolem, Chabauty and Coleman for studying diophantine equations. From model theory we use an uniform statement due to Scanlon for the Manin–Mumford problem on the additive group scheme.

BRADD HART, McMaster University
Group existence in simple continuous theories

This is a preliminary report on joint work with Jean-Martin Albert on the existence of definable groups in simple, first-order continuous, theories.

DIERDRE HASKELL, McMaster University, 1280 Main St. W, Hamilton, ON, L8P 2T4
Stable domination and algebraically closed valued fields

The concept of stable domination has been developed as a way to extend the tools of stability theory to structures which are not stable but have a rich stable reduct (in a precise sense). In this talk, I will define stable domination and its associated notion of domination independence, and illustrate how some standard properties of independence in a stable theory lift to corresponding properties of stably dominated types. I will illustrate these properties with examples in algebraically closed valued fields.

IVO HERZOG, The Ohio State University, 4240 Campus Dr., Lima, OH 45804
Definable subspaces of finite dimensional representations

Let k be an algebraically closed field of characteristic 0, and denote by L a finite-dimensional semisimple Lie algebra over k. If $\phi(v)$ is a positive-primitive formula in the language of modules over the universal enveloping algebra $U(L)$, then the subset $\phi(V)$ defined by ϕ in an L-representation V is a subspace of V, considered as a vector space over k. If $\phi(V)$ defines a sum of weight spaces of V, for every finite-dimensional representation V, then there is a positive-primitive formula ϕ^- that defines an orthogonal complement of $\phi(V)$, for every finite-dimensional V. The talk will be devoted to a proof of this fact, as well as an explanation of the conjecture that if $\phi(V)$ defines the 1-simplex of weights whose boundary consists of the highest weight space, and one of its conjugates under a simple reflection, then $\phi(V)$ must be a minimal linearly bounded formula.

THOMAS KUCERA, Department of Mathematics, University of Manitoba
Elementary Socles and Radicals

Socles and radicals are important tools in studying the structure of modules and rings. The socle of a module is the sum of all of its simple (minimal) submodules; dually the radical of a module is the intersection of all of its maximal submodules. Ivo Herzog introduced model-theoretic analogues of these concepts by replacing "submodule" by "definable subgroup".

In an (indecomposable) totally transcendental module the elementary socle is non-trivial and is a definably closed submodule. Furthermore, the definition of elementary socle naturally extends to an ascending series of definably closed submodules whose union is the whole module. Dually, if an indecomposable pure-injective module has the ascending chain condition on definable subgroups (ACC-pp), the elementary radical is a proper submodule, and the definition of the elementary radical may be extended to a descending series of submodules whose intersection is 0.

Mike Prest introduced a notion of duality between certain first order formulas in the languages of left modules and right modules which Herzog extended to a duality of categories. This duality makes indecomposable tt modules correspond to indecomposable pure-injective modules with ACC-pp. I show that there is a natural similarity between the structure of the elementary socle series of an indecomposable tt module and the structure of the elementary radical series of its elementary dual.

The elementary socle series has had limited application in describing the structure of certain indecomposable injective modules; however serious applications await a deeper understanding of the properties of these series in general.

SALMA KUHLMANN, University of Saskatchewan, 106 Wiggins Road, Saskatoon, SK S7N 5E6
Restricted Exponentiation in Fields of Algebraic Power Series

Ron Brown (1971) proved that a valued vector space of countable dimension admits a valuation basis. This result was applied by S. Kuhlmann (2000) to show that every countable real closed field admits a restricted exponential function, that is, an order preserving isomorphism from the ideal of infinitesimals $(\mathcal{M}_K, +)$ onto the group of 1-units $(1 + \mathcal{M}_K, \cdot)$. A natural question arose whether every real closed field admits a restricted exponential function. In this talk, we give a partial answer to this question. To this end, we investigate valued fields which admit a valuation basis (as valued vector spaces over a given countable ground field K). We isolate a property (called TDRP in our paper) for a valued subfield L of a field of generalized power series $F((G))$ (where G is a countable ordered abelian group and F is a real closed, or algebraically closed field). We show that this property implies the existence of a K-valuation basis for L. In particular, we deduce that the field of rational functions $F(G)$ (the quotient field of the group ring $F[G]$) and the field $F(G)$ of power series in $F((G))$ algebraic over $F(G)$ admit K-valuation bases. If moreover F is archimedean and G is divisible, we conclude that the real closed field $F(G)$ admits a restricted exponential function.

CHRIS LASKOWSKI, University of Maryland
Ideals of formulas, 2-cardinal models, and group existence

We survey a host of results involving ideals of formulas, both inside and outside the context of stability. In some cases we construct 2-cardinal models, while in others we assert the existence of definable groups.

RAHIM MOOSA, University of Waterloo, Waterloo, Ontario, N2L 3G1
A consequence of the canonical base property

A stable theory of finite rank is said to have the Canonical Base Property (CBP) if for any stationary type p, the type of the canonical base of p over a realisation of p is (almost) internal to the collection of all non-modular minimal types. No examples of the failure of CBP are known. Inspired by the model theory of compact complex manifold, but working in an arbitrary complete stable theory T of finite rank with the CBP, we give a geometric characterisation of when a stationary type is internal to the collection of all non-modular minimal types. Our characterisation is based on, and partially recovers, Campana's "second algebraicity criterion" from complex analytic geometry.

This is joint work with Anand Pillay.

PHILIPP ROTHMALER, CUNY, University Ave & W 181 Street, Bronx, NY 10453

Cotorsion modules

Cotorsion modules represent a homologically defined generalization of pure-injective modules. There are cotorsion abelian groups that are not pure-injective, but over von Neumann regular rings (that is, rings over which all modules have quantifier elimination) and over pure semisimple rings (that is, rings over which every module is totally transcendental), this generalization yields nothing new. Joint work with Ivo Herzog will be presented that characterizes the class of rings (containing the former two classes) over which every cotorsion module is pure-injective.

THOMAS SCANLON, University of California Berkeley, Department of Mathematics, Evans Hall, Berkeley, CA 94720-3840, USA

Trivial types and rational dynamics

Recall that a periodic point of a polynomial $f(x) \in \mathbb{C}[x]$ is a complex number a for which $f^m(a) = a$ for some positive integer m. We discuss how for certain choices of polynomials the algebraic relations amongst the periodic points may be understood through the study of associated trivial types in the theory of difference fields.

PATRICK SPEISSEGGER, McMaster University, 1280 Main St. W, Hamilton, ON, L8S 4K1

A reasonably tame Cantor set

We construct a Cantor set $E \subset [0,1]$ such that for every $n \in \mathbb{N}$ and every bounded $f : A \longrightarrow \mathbb{R}^m$ definable in any polynomially bounded o-minimal expansion of the real field, the image $f(E^n \cap A)$ is Minkowski null. It follows that the expansion of the real field by E does not define the set of all natural numbers.

Joint work with Harvey Friedman, Krzysztof Kurdyka and Chris Miller.

YEVGENIY VASILYEV, University of Windsor, Department of Mathematics and Statistics, Windsor, ON, N9B 3P4

Externally definable sets in simple structures

I will talk about the issue of quantifier elimination in the expansion of a simple structure with the traces of relations definable in some elementary extension. By Shelah's result, in the case of theories without the independence property, q.e. holds if we add all such traces (with parameters from a saturated extension). Since q.e. fails if we apply the same procedure to a random graph, in the simple case we restrict to parameters coming from a "lovely pair" extension. In this setting, in a joint work with Anand Pillay, we show that the expansion having q.e. is equivalent to a certain definability condition (weak lowness) on the base theory, and find an example of a non-weakly low simple theory. I will also discuss other possible ways of adding externally definable sets, such as expansions by "generic" traces.