THOMAS KUCERA, Department of Mathematics, University of Manitoba
Elementary Socles and Radicals

Socles and radicals are important tools in studying the structure of modules and rings. The socle of a module is the sum of all of its simple (minimal) submodules; dually the radical of a module is the intersection of all of its maximal submodules. Ivo Herzog introduced model-theoretic analogues of these concepts by replacing "submodule" by "definable subgroup".

In an (indecomposable) totally transcendental module the elementary socle is non-trivial and is a definably closed submodule. Furthermore, the definition of elementary socle naturally extends to an ascending series of definably closed submodules whose union is the whole module. Dually, if an indecomposable pure-injective module has the ascending chain condition on definable subgroups (ACC-pp), the elementary radical is a proper submodule, and the definition of the elementary radical may be extended to a descending series of submodules whose intersection is 0.

Mike Prest introduced a notion of duality between certain first order formulas in the languages of left modules and right modules which Herzog extended to a duality of categories. This duality makes indecomposable tt modules correspond to indecomposable pure-injective modules with ACC-pp. I show that there is a natural similarity between the structure of the elementary socle series of an indecomposable tt module and the structure of the elementary radical series of its elementary dual.

The elementary socle series has had limited application in describing the structure of certain indecomposable injective modules; however serious applications await a deeper understanding of the properties of these series in general.