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Description of the injective modules over a principal left and right ideal domain

Over a principal left and right ideal domain R every injective module is a direct sum of indecomposable injective modules. One indecomposable injective is the injective envelope (divisible hull) of the module ${}_RR$ and is isomorphic to the division algebra Q of R. The other indecomposable injective modules are (up to isomorphism) in a one-to-one correspondence with the prime elements of the ring (up to similarity).

Motivated by a classic treatment of O. Ore, I take advantage of the factorization theory in R and investigate the internal structure of an indecomposable injective left module $E \neq Q$. I describe its "layered" structure in terms of its elementary socle series $\left(\mathrm{soc}^{\alpha}(E)\right)_{\alpha}$, a concept which was introduced by I. Herzog as the elementary analogue of the socle series of a module, where the minimality condition on the pp-definable subgroups is used. Since E has the descending chain condition on pp-definable subgroups, the elementary socle series exhausts E. A complete characterization of $\left(\mathrm{soc}^{\alpha}(E)\right)_{\alpha}$ is obtained.

In addition, I will analyze the relationship between the classical socle series of the right module E over the ring $T = \mathrm{End}_R(E)^{\mathrm{op}}$ and the elementary socle series of the left R-module E.