Mathematical Physics
Physique mathématique
(Org: **Richard Froese** (UBC) and/et **Tom Osborn** (Manitoba))

TWAREQUE ALI, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec, Canada, H3G 1M8
Berezin–Toeplitz operators and vector coherent states related to supersymmetric quantum mechanics

In this talk we intend to discuss a set of newly constructed vector coherent states and associated Berezin–Toeplitz operators, which arise in supersymmetric quantum mechanics. Assuming that supersymmetry is unbroken, these coherent states are related to the two almost isospectral (i.e., except for the ground state) Hamiltonians of the theory. The analytic representation of the system lives on a Hilbert space consisting of two mutually orthogonal subspaces of analytic and anti-analytic functions, corresponding to the bosonic and fermionic sectors, respectively. The representation can also be transferred to a supersymmetric Hilbert space. We discuss the associated quantization problem and the ensuing Berezin–Toeplitz operators.

RAINER DICK, University of Saskatchewan, Department of Physics
String and brane techniques for surface science and nanotechnology

String theory is a well-known catalyst for seminal interactions between mathematics and physics. Theoretical physicists became aware of powerful mathematical tools like Lie algebra cohomology or the theory of complex manifolds when they had to use these methods to understand the quantum dynamics of relativistic strings.

It is well known that some of the mathematical techniques used in string theory also apply to two-dimensional critical systems, but it is not so widely known that some of the mathematical techniques used in brane world theories can also be adapted for calculations in low-dimensional electron systems.

The dynamics of electrons at surfaces, interfaces, and quantum wires is important for materials science and for the investigation of magnetic and thermodynamic properties of low-dimensional systems.

The talk will specifically discuss the mapping between half-order differentials and spinors for the description of fermions in low-dimensional systems, and the use of brane world techniques to describe the transition between surface and bulk properties of electrons in the presence of a surface or interface.

RICHARD FROESE, University of British Columbia
Absolutely continuous spectrum for discrete Schrödinger operators on tree like graphs

I will dicuss results obtained with David Hasler and Wolfgang Spitzer on absolutely continuous spectrum on graphs, including our alternative proof of Klein's result on extended states for the Anderson model on the Bethe Lattice.

STEPHEN GUSTAFSON, University of British Columbia
Schroedinger Flow and Landau-Lifshitz Dynamics

The Schroedinger flow into the 2-sphere is a geometric generalization of the linear Schroedinger equation, and a particular case of the Landau–Lifshitz equation for ferromagnets. Understanding the fate of solutions (blow-up? asymptotics?) is a challenge. I will describe some recent work on these questions, part of a joint project with K. Kang and T.-P. Tsai.

NAN-KUO HO, National Cheng-Kung University, Department of Mathematics, Tainan City, 701, Taiwan
Yang–Mills connections on 2-manifolds

In "The Yang–Mills equations over Riemann surfaces", Atiyah and Bott studied the Yang–Mills functional over a Riemann surface from the point of view of Morse theory. We generalize their study to all closed, compact, connected, possibly nonorientable surfaces. We introduce the notion of "super central extension" of the fundamental group of a surface. It is the central extension when the surface is orientable. We establish a precise correspondence between Yang–Mills connections and representations of super central extension. Knowing this exact correspondence, we work mainly at the level of representation varieties which are finite dimensional instead of the level of strata which are infinite dimensional.

I will explain the Yang–Mills functional and Yang–Mills connections over a Riemann surface and compare with the nonorientable surface case that we studied.

This is a joint work with C.-C. Melissa Liu.

EUGENE KRITCHEVSKI, McGill University, Montreal
Hierarchical Anderson Model

The hierarchical Anderson model is the random self-adjoint operator

$$H = L + cV,$$

where L is a hierarchical Laplacian, V is a random potential and $c > 0$ is a coupling constant measuring the strength of the disorder. In this talk, I will first review the basic properties of L and the associated spectral dimension d. Then I will present the following results about the spectral behavior of H.

(1) If $d < 4$ then, with probability one, the spectrum of H is pure point at all energies and for all c.

(2) If $d < 1$ then, in a natural scaling limit, the eigenvalues of finite volume approximations to H converge to a Poisson point process.

KARL-PETER MARZLIN, Institute for Quantum Information Science, University of Calgary
Criteria for the Existence of Decoherence-free Subspaces

Decoherence-free subspaces (DFS) are spanned by such states of an open quantum system that are insensitive to the decoherence induced by the reservoir to which the system is coupled. DFS are immune to this coupling because of different physical effects, including destructive interference between different transition amplitudes or energy conservation.

We compare different definitions of DFS and explore rigorous criteria for the existence of DFS in finite-dimensional systems coupled to Markovian reservoirs. The advantages and disadvantages of various approaches are compared and a geometrical interpretation for DFS in qubit-systems is given.

MARCO MERKLI, Memorial University
Infinite Products of Random Matrices and Repeated Interaction Dynamics

Let Ψ_n be a product of n independent, identically distributed random matrices M, with the properties that Ψ_n is bounded in n, and that M has a deterministic (constant) invariant vector. Assuming that the probability of M having only the simple eigenvalue 1 on the unit circle does not vanish, we show that Ψ_n is the sum of a fluctuating and a decaying process. The latter converges to zero almost surely, exponentially fast as $n \to \infty$. The fluctuating part converges in Cesaro mean to a limit that is characterized explicitly by the deterministic invariant vector and the spectral data of $\mathbb{E}[M]$ associated to 1. No additional assumptions are made on the matrices M; they may have complex entries and not be invertible.

We apply our general results to two classes of dynamical systems: inhomogeneous Markov chains with random transition matrices (stochastic matrices), and random repeated interaction quantum systems. In both cases, we prove ergodic theorems for the dynamics, and we obtain the form of the limit states.

ROB MILSON, Dalhousie University

Curvature homogeneous geometries

A pseudo-Riemannian manifold M is curvature homogeneous of order s if the components of the curvature tensor and its first s covariant derivatives are constant relative to some local frame. If M is locally homogeneous, then it is curvature homogeneous. Remarkably, the converse is also true, in some fashion: if M is curvature homogeneous of order s and if s is greater than a certain bound, called the Singer index, then M is locally homogeneous. We establish that the Singer index for 4-dimensional CH, Lorentzian manifolds is equal to 2. Our approach is to formulate the field equations for a CH geometry as an involutive EDS on the second-order frame bundle of M.

DAVID ROWE, Department of Physics. University of Toronto, Toronto, ON, M5S 1A7

Quasi-symmetry, critical phenomena, and embedded representations

The use of symmetry in the description of physical systems has turned up a novel type of representation in group theory that has deep implications for understanding critical phenomena. Loosely speaking, if one is given a unitary representation of a group on a Hilbert space, it can turn out that the projection of this representation onto a subspace may be another unitary representation that is neither a subrepresentation nor a subquotient of the original; such a representation is called an *embedded representation*. This concept provides a natural framework for understanding why transitions between phases of systems, associated with different symmetries, frequently exhibit critical phenomena. It is observed that a system in one phase, appears to hold onto the symmetry associated with that phase until a breaking point is reached at which a rapid transition occurs to a new phase associated with a different symmetry. In fact, it appears that such an apparent symmetry, which we call a quasi-symmetry, is appropriately associated with an embedded representation that can change continuously as a system approaches a critical point.

CRISTINA STOICA, Wilfrid Laurier University

Variational principles for systems with configuration space isotropy

This presentation considers Lagrangian systems on tangent bundles, with lifted symmetries and configuration space isotropy. We use a twisted parametrisation of the phase space corresponding to phase space slices based at zero points of tangent fibres. Using Hamilton's variational principle with appropriate constraints, we deduce the Lagrangian bundle equations in the twisted coordinates. This complements earlier work describing the dynamics on the cotangent side.